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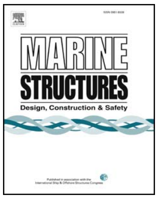

Semi-analytical model for the prediction of the post-buckling behaviour of unstiffened cylindrically curved steel panels under uniaxial compression

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ABSTRACT

Owing to its structural efficiency stiffened steel curved panels are increasingly used in structural applications in civil, naval and offshore engineering. However, design provisions to predict their strength are practically non-existent. Consequently, the aim of this paper is to present a robust semi-analytical procedure for the prediction of the post-buckling behaviour of unstiffened cylindrically curved steel panels under uniform uniaxial compression. Nonlinear stability models based on large deflection theory incorporating initial imperfections and geometric nonlinearity are implemented. The problem is solved through the Rayleigh-Ritz method and the post-buckling solutions are obtained in order to assess local buckling of isotropic panels. Two distinct simply supported boundary conditions are considered, depending on the in-plane restraints of edges. The validity of these computational models is assessed with the results of finite element analyses for different curvatures and aspect ratios, yielding good results. Finally, design-oriented closed-form analytical expressions are derived based on single degree of freedom approximations.

1. Introduction

Owing to their structural efficiency, curved steel panels are increasingly used in civil, naval and offshore applications [1]. In spite of being highly susceptible to local and global instabilities, these elements present a good overall structural efficiency. However, there is a lack of robust and physically consistent design formulations, both in construction and offshore standards, to deal with all the variabilities inherent to these panels. The aim of this paper is to present a semi-analytical procedure for the prediction of the elastic post-buckling behaviour of isotropic unstiffened cylindrically curved steel panels under uniform uniaxial compression. The formulation is based on the classical shell theory with large deflection theory and geometric nonlinearity (von Kármán-Donnell kinematic nonlinearity) incorporating initial imperfections [2]. The equilibrium solutions are obtained using the Rayleigh-Ritz method. Two distinct cases of boundary conditions are considered. Both are simply supported with the transversal loaded edges kept straight. However, in one case the longitudinal unloaded edges are kept straight (C) and in the other they are free to displace in-plane (U).

The validity of the formulation is assessed by comparing the obtained results using a multi degree of freedom displacement field with elastic large displacement finite element results using shell elements, for a wide range of curvatures and aspect ratios. Finally, a design-oriented analytical single degree of freedom solution is presented that allows for closed-form design expressions.

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2. State of the art

The study of large-displacement theory for curved elements started with thin cylindrical tubes (closed shells) by Donnell [3] and von Kármán and Tsien [4]. The authors concluded that the classical theory of thin shells of perfect elements (see Southwell [5]) is inadequate to explain the intricate behaviour of cylindrical shells (the calculated buckling load can be several times higher than the maximum load found by experiments). Various authors, as Leggett [6], Michielsen [7] and Almroth [8], extended those studies by incorporating more terms in the displacement function to improve the agreement of the theoretical solutions with experimental results. Addressing the study of pressurized cylindrical shells under axial compression, Hutchinson [9] explicitly incorporated initial imperfections, which were found to have an important influence.

The interest in curved panels (a part of a cylindrical shell) started with their use in the fuselage of airplanes. The study of curved panels was started in the 40s by Levy [10] by adapting the large-deflection theory of flat plates for panels with an initial curvature. Through a simplified consideration of the curvature, the author assumed a curved strip plate with an initial sinusoidal displacement in one direction only. The panels were considered free from imperfections and simply supported along the edges parallel to the generator. The author concluded that curvature may cause an important increase in the buckling load. Based on a modified form of Donnell's equations, Batdorf [11] studied the buckling stresses of simply supported and clamped perfect cylindrically curved panels using trigonometric series to approximate the displacements. The author claimed that these modified equations were better adapted to solutions by Fourier series for both boundary conditions. Volmir [12], adopting the Galerkin method, proposed an approximate solution for the post-buckling behaviour of perfect thin curved panels. However, the boundary conditions were not fully satisfied and good accuracy could not be achieved. To deal with this problem, Tamate and Sekine [13] proposed an improved solution for the post-buckling behaviour of simply supported perfect thin curved panels with all edges simply supported and free to deflect. Zhang and Matthews [14] studied the behaviour of perfect shallow cylindrical curved panels of layered composite materials using the principle of virtual displacements. Recognizing that imperfections might not be the only reason for discrepancy with experimental tests, some authors also started including the effect of edge restraints in compressed curved panels. Based on an elastic foundation analogy, Chia [15] investigated the nonlinear vibration and post-buckling behaviour of laminated imperfect curved shallow panels with edges elastically restrained against rotation.

Chang and Librescu [16] studied the post-buckling of imperfect shear deformable doubly curved shallow panels under compressive and lateral pressure using a shear deformation theory (SDT) which is of direct relevance to laminated materials. They adopted the Galerkin method to derive the post-buckling response of curved panels with: *i)* edges simply supported and freely movable and *ii)* simply supported edges and longitudinal edges unloaded and rigidly held apart. Breivik [17] investigated the post-buckling behaviour of unstiffened curved composite panels subjected to combinations of thermal and mechanical end-shortening loading. The author used the Rayleigh-Ritz method assuming the displacement functions for all directions, avoiding to solve the fourth-order von Kármán-Donnell differential equations. Clamped boundary conditions were assumed along the curved edges and either sliding or fixed boundary conditions were assumed along the longitudinal edges. Shen [18] used a higher order shear deformation theory (HSDT) with a von Kármán-Donnell type of kinematic non-linearity for axially loaded shear-deformable laminated curved panels. In order to obtain the post-buckling solutions a singular perturbation technique was employed. Watamori and Kasuya [19] assessed the post-buckling behaviour of laminated imperfect curved panels under axial compression based on the second variation of the potential energy. Wilde et al. [20] studied the buckling stresses of cylindrical curved panels with three edges simply supported and one free subjected to axial compression. Duc and Tung [21] investigated functionally graded imperfect curved panels (with a material made from aluminium and ceramic) under axial compression using the Galerkin method. All edges were considered simply supported but in-plane restraints were not considered. Blandzi and Magnucki [22] studied the buckling and post-buckling of cylindrical curved panels under compression using the Galerkin method. Imperfections were not considered and the boundary conditions were assumed as simply supported but once again the in-plane restraints at the edges were not taken into account. Martins et al. [23] studied the critical behaviour of unstiffened cylindrically curved panels under in-plane stresses. Based on energy methods, the authors developed expressions for the elastic critical stresses for simply supported panels considering the non-loaded longitudinal edges restrained and free to deflect. Displacement functions were assumed for each direction obviating the solution of the differential equations. The number of degrees of freedom was found to be important.

The use of semi-analytical methods to investigate the post-buckling behaviour of flat panels is documented in the literature [24] [25], whereby the problem is formulated analytically and solved, but some coefficients are obtained from the numerical solution of a differential equation. In the context of the post-buckling behaviour of flat plated using semi-analytical methods, Byklum and Amdahl [26] recently implemented an arc-length method able to deal with snap-through and snap-back problems and Ferreira and Virtuoso [27] studied the influence of distinct simply supported boundary conditions.

To the best of the authors' knowledge, the solution of the problem as presented in this paper is not available in the literature. In fact, the behaviour of curved panels is very complex and a rigorous solution requires the solution of boundary value problems of fourth order nonlinear partial differential equations. For that purpose, the Airy's function must contain the effect of edge restraint and at the same time verify the von Kármán-Donnell equations. A multi degree-of-freedom (MDOF) solution accounting for constrained or unconstrained simply supported boundary conditions (with transversal loaded edges forced to remain straight and longitudinal non-loaded edges forced to remain straight or free to deflect) with imperfections included and applying the Rayleigh-Ritz method was not found in the literature and it is presented in detail in the following sections.