NDT and E International 96 (2018) 1–8

Contents lists available at ScienceDirect

NDT and E International

journal homepage: www.elsevier.com/locate/ndteint

Solution for a transmit-receive eddy current probe above a layered planar conductive structure

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ARTICLE INFO

Keywords:
Eddy current testing
Multi-frequency
Nondestructive testing
CANDU®
Transmit-receive probe
Layered
Planar
Analytic solutions
Boundary value problem

ABSTRACT

Exact solutions for the electromagnetic response of a transmit-receive coil pair situated above two parallel plates that are separated by a gap, have been developed. The analytical model was used to characterize the effect of variations in probe liftoff, conductor resistivity, plate wall thickness, and plate-to-plate gap. All electromagnetic coupling coefficients arising from the probe and layered plate conductors were determined and substituted into Kirchhoff's circuit equations to calculate the induced voltage in the pickup coil. Excellent agreement with experiment was observed for differing first layer plate resisitivities (174 μΩ cm and 53.9 μΩ cm), first layer plate thicknesses (3.80 mm and 3.18 mm), second layer plate resistivity (1.72–174 μΩ cm), second layer plate wall thickness (1.20 mm–4.85 mm) and probe liftoff (2.81 mm and greater), providing confidence in the general validity of the model.

1. Introduction

The motivation for this work was to develop an analytical model to simulate the response of the Transmit-Receive (TR) Eddy Current (EC) probe used by Ontario Power Generation (OPG) [1,2] to monitor the gap between the Pressure Tubes (PTs) and Calandria Tubes (CTs) (within which the PTs are contained) of CANDU® fuel channels. The PT-CT gap requires monitoring to avoid contact conditions, which may result in delayed hydride cracking of the PT [3,4]. Although EC testing using a TR probe (two horizontally offset coils) is effective for in-reactor inspection, multiple experimental parameters such as probe liftoff, PT resistivity, and Wall Thickness (WT) can affect the EC-based PT-CT gap measurement [1]. While PT WT can be measured by ultrasonic techniques [5], probe liftoff, and PT resistivity cannot be directly measured by existing experimental procedures and may cause systematic errors in the gap measurement. The development of a rigorous mathematical model is a first step in developing techniques to perform simultaneous multi-parameter measurements and to assist in interpreting inspection data [3,6].

Towards this end, Shokralla et al. [1,2] recently developed a mathematical model to simulate the in-service EC probe. The radius of the PT (nominally 51 mm) was large relative to the coil-to-coil spacing (~11 mm) and its relative position above the inner diameter of the PT (~2 mm) [1]. For simplicity, the PT and CT were approximated as parallel flat-plates, separated by an air gap [1]. Shokralla et al. [1] confirmed the validity of this approximation on the total amplitude response, measured experimentally, for excitation frequencies greater than 4 kHz. The flat-plate equivalents of the PT and CT are denoted as the Near-Plate (NP) and Far-Plate (FP), respectively. In addition, the model of Shokralla et al. [1,2] approximated the TR configuration as a series of axially-concentric coils. The pickup coil was approximated by two concentric pickup coils, with cross-sectional areas overlapping the cross-sections of the actual pickup coil [1]. The signal difference between the concentric pickup coils was calculated, and was normalized by the ratio of the coil-pair volume to the volume of the actual pickup coil [1]. In addition, the solutions generated by Shokralla et al. [1] used the general model of Dodd and Deeds [7–9], which is only valid for a constant amplitude current excitation of the drive coil and open-circuit pickup coils. This is problematic because the drive coil is excited by a constant amplitude voltage source and in this case, the drive coil current itself is affected by the presence of the conducting components (see Section 2.1).

The intrinsic limitations of Shokralla et al.'s model [1] have been overcome by the exact analytical solutions developed in this paper. This was achieved by using a more general model derived by Desjardins et al. [10,11], which accounts for all feedback effects present for a voltage-controlled probe with finite coil impedances in combination with exact solutions for a TR probe geometry. This new model was validated

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https://doi.org/10.1016/j.ndteint.2018.02.003
Received 7 June 2016; Received in revised form 7 July 2017; Accepted 19 February 2018
Available online 21 March 2018
0963-8695/

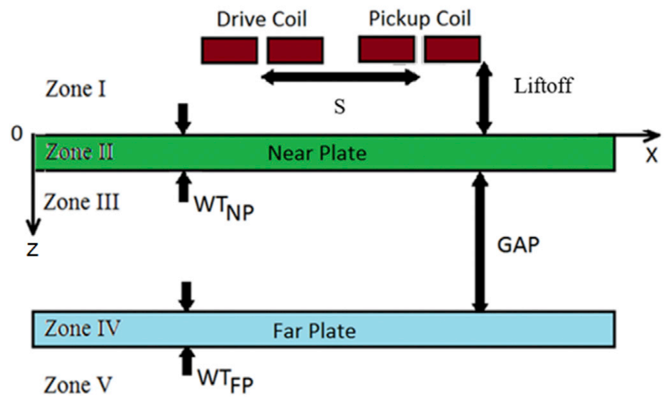

Fig. 1. Cross-sectional view of the eddy current probe above a conducting layered structure. Zone I is air, Zone II is the near plate, Zone III is the air gap between near plate and far plate, Zone IV is the far plate and Zone V is air. *S* is the coil-to-coil spacing, while WT_{NP} and WT_{FP} are the near and far plate wall thicknesses, respectively.

with experimental data over a broad range of tested physical parameters. This rigorous validation further supports the generality of the voltage-controlled model developed by Desjardins et al. [10,11] as opposed to the widely-used model of Dodd and Deeds [7–9]. In Ref. [12], Dodd and Deeds [7–9] constant amplitude alternating current approximation applied to the probe geometry was investigated in Ref. [12] and was found to not represent experimental data, if a large drive coil self-inductance and high frequency excitations were present. This work presents the analytical and experimental results used for validation of the analytical model, both of which were not presented in Ref. [12].

A probe was manufactured with similar specifications as the probe described in Refs. [1,2]. However, the methodology discussed in this work can be applied to any TR probe, and the reader is directed to Refs. [7,13,14] for guidance in choosing appropriate probe dimensions for a given application. A cross-section of the TR probe and the approximated fuel channel geometry is shown below in Fig. 1.

2. Theory

2.1. Overview of the general model

A time-harmonic voltage is applied to the drive coil, which generates a time-varying magnetic field and induces eddy currents in the nearby conducting structures [11]. From Faraday's law, an electromotive force (emf) is induced in the drive and pickup coils from the changing magnetic flux arising from the drive coil source current. As shown in Fig. 2,

Table 1
The solutions to Eqn. (9) in each zone (see Fig. 1) for a particular coil.

Zone	Eddy Current Impulse Response Function	
I	$\widehat{\boldsymbol{A}}_{d,p}(\gamma,z,\omega)=B_I^{(1)}(\gamma,\omega)e^{\gamma z}+\widehat{\psi}_{d,p}(\gamma,z,\omega)$	(12)
II	$\widehat{\boldsymbol{A}}_{d,p}(\gamma,z,\omega)=B_{II}^{(1)}(\gamma,\omega)e^{\sqrt{\mu_0\sigma_{NP}j\omega+\gamma^2}z}+B_{II}^{(2)}(\gamma,\omega)e^{-\sqrt{\mu_0\sigma_{NP}j\omega+\gamma^2}z}$	(13)
III	$\widehat{\boldsymbol{A}}_{d,p}(\gamma,z,\omega)=B_{III}^{(1)}(\gamma,\omega)e^{\gamma z}+B_{III}^{(2)}(\gamma,\omega)e^{-\gamma z}$	(14)
IV	$\widehat{\boldsymbol{A}}_{d,p}(\gamma,z,\omega)=B_{IV}^{(1)}(\gamma,\omega)e^{\sqrt{\mu_0\sigma_{FP}j\omega+\gamma^2}z}+B_{IV}^{(2)}(\gamma,\omega)e^{-\sqrt{\mu_0\sigma_{FP}j\omega+\gamma^2}z}$	(15)
V	$\widehat{\boldsymbol{A}}_{d,p}(\gamma,z,\omega)=B_V^{(1)}(\gamma,\omega)e^{-\gamma z}$	(16)

Table 2
List of interfaces (defined in Fig. 1) with boundary conditions for equations given in Table 1.

Interface	Position *z*	
I-II	$z=0$	(19)
II-III	$z=WT_{NP}$	(20)
III-IV	$z=WT_{NP}+GAP$	(21)
IV-V	$z=WT_{NP}+GAP+WT_{FP}$	(22)

there are up to six modes of inductive coupling present in the configuration of a transmit-receive probe above a conducting plane, when the drive and pickup coils are assumed not to be in an open-circuit configuration and the drive coil is excited by a voltage source [10,11]. These forms of electromagnetic coupling are mathematically described as the self-inductance *L* and mutual inductance *M* of the probe [11,15]. The self and mutual inductances are only dependent on the coil dimensions and

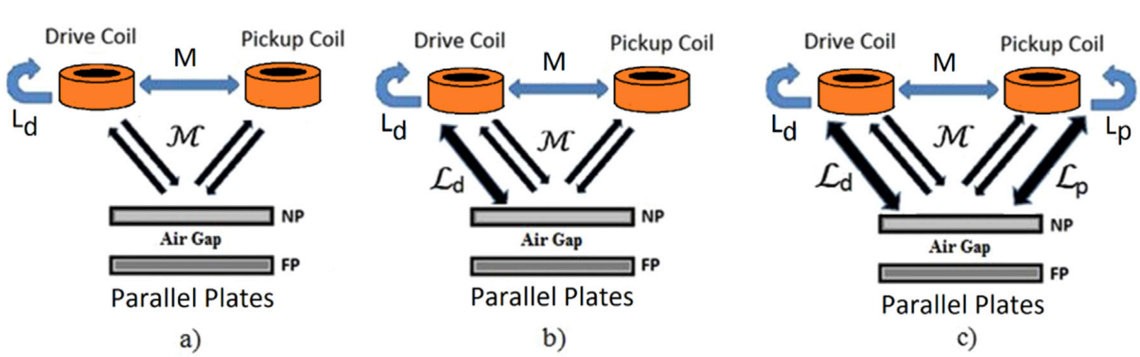

Fig. 2. A visual presentation showing all electromagnetic interactions as addressed by (a) Dodd and Deeds' [7–9] with the constant amplitude drive coil current approximation, (b) Dodd and Deed's model without the constant amplitude drive coil current approximation (or Desjardins et al. [10,11] with open-circuit pickup coils, and c) the general model of Desjardins et al. [16] NP is the near plate and FP is the far plate.