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Scattering of guided shear waves in plates with discontinuities

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ABSTRACT

This paper investigates the interaction of the SH0 mode with discontinuities in plate waveguides. The scattered fields are evaluated using a novel method that exploits the principle of reciprocity in elastodynamics. The results obtained compare to those provided by a finite element model. Very good agreement between the analytical and numerical models proves the effectiveness of the proposed approach, enabling us to clearly elucidate the role of the different size and shape parameters involved. The discontinuities considered are single and double sharp reductions of plate height of different amounts and lengths, where both symmetric and nonsymmetric one-sided notch cases are treated. Regimes related to low and high values of the product frequency and height of the plate are investigated, showing the dependence of reflection and transmission coefficients on length and height of the discontinuity, as well as the occurrence of mode conversion. The analytical approach proposed leads to a better understanding of the interaction of guided waves with discontinuities, which may stimulate the application of guided waves to defect sizing rather than to simple detection.

1. Introduction

In the past few decades, guided waves have played an important role in nondestructive health monitoring, with applications ranging from the detection of cracks and corrosion to the monitoring of states of stress [1,2]. The success of guided waves is related to the geometric waveguide structure of many structural elements such as beams, rails, and pipes. With remarkable advantages as compared to bulk waves for inspection areas, guided waves propagating in such solids can be used to monitor large structural portions thanks to the existence of modes with minimal attenuation. This technique has been mainly used for defect screening rather than for defect characterization because of the many difficulties that arise when the scattered field that originated from a wave encountering a discontinuity has to be interpreted.

Practical guided-wave ultrasonic testing is done by sending a signal along a waveguide and interpreting the scattered response. In the simplest case, one single-mode signal is used. In the presence of a defect, the transmitted and reflected responses consist of a complex superposition of wave modes. At any frequency, many modes exist and the incident wave can be converted into a multimode reflected or transmitted signal in order to satisfy boundary conditions. These modes are often dispersive, and so the shape of the multimode signal can change with distance and the resulting pattern can be rather complex [3,4].

Better knowledge of wave interaction with defects can be of use in different applications, in particular in the investigation of solutions to the inverse problem of defect characterization based on the response variation. The ability to describe the variation of scattering coefficients as a function of the geometric characteristics of the discontinuity is fundamental for the evaluation of the uniqueness of the solution to the inverse problem. Recently, such ability has given rise to the application of guided waves to defect sizing and shape reconstruction of surfaces and inner defects [5, 6, 7–9]. The knowledge of wave interaction with defects can also help in the selection of modes and frequencies that improve inspection sensitivity to various discontinuities.

Determining reflections and transmission coefficients from discontinuities of different kinds is challenging. Several approaches have been proposed. Many researchers have investigated this topic, and a complete review of the vast literature is beyond the scope of this paper. In general, it is necessary to resort to numerical methods. The different approaches can be classified as methods based on wave expansion, often referred to as mode matching or modal decomposition methods [10–14], finite element methods [3,4,15–18], or hybrid numerical methods combining finite element formulations with waves or boundary elements [19–21].

The approach used in this paper belongs to the category of modal decomposition methods, which use integral formulations to enforce boundary conditions. Reflection and transmission coefficients are obtained using a method previously proposed by the

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authors [22] based on the principle of reciprocity in elastodynamics [23,24], which relates the elastic solutions of two different loading states. Our approach exploits the vector projection deriving from the principle of reciprocity that always preserves the energetic equivalence between power fields, independently of the number of modes considered. Poddar and Giurgiutiu [26] have independently just come up with something analogous by simply projecting the boundary conditions, which, however, requires a higher number of modes for convergence. The idea of connecting two states with reciprocity to determine scattering coefficients was developed first for bulk wave propagation [25] and then for waveguides [11]. Analogously, here, a virtual or test wave whose solution is known is used to obtain information on another elastodynamic state, which is the response of the waveguide in the presence of the discontinuity. Differently from Ditri [11], here the analysis is not limited to a crack, but, using double sharp discontinuities, we assume that the defect has a finite extension that sustains wave modes. In such a way, multiple localized vertical discontinuities, be they symmetric or antisymmetric, can be dealt with.

The focus of this study concerns discontinuities with simple shapes, which are enough to present some interesting aspects of the scattering. The shapes are sharp single- or double-step changes of cross section, which in real structures apply to notches, joints, or corroded areas. In particular, the interaction of the shear SH0 wave mode with such changes in the height of a plate is studied. Symmetric and nonsymmetric notch cases are both considered. Some aspects of the problem treated in this paper were investigated in Demma et al. [17], who limited their analysis to low frequencies (no mode conversion), and in Song et al. [21], who reported experiments demonstrating the sizing capabilities of tests conducted with an SH0 shear wave in a plate with overlap. It is also worth recalling Rajagopal and Lowe [3] and Ratassep et al. [4], where a finite element model (FEM) of a plate was used to show the diffraction of the SH0 wave due to through-thickness cracks.

Shear waves are chosen because they enable a simple comparison between the reflection and transmission coefficients calculated using the proposed approach, which gives an analytical solution, and the results obtained from FEM. Moreover, by changing the height of the plate, situations in which single or multiple modes are expected in the scattered fields can be investigated, and the phenomenon of mode conversion can be observed. From a practical point of view, the SH0 mode is nondispersive and can be applied to both plates and pipes because its dynamics also satisfactorily describe the behavior of the first torsional mode in pipes of large radius [17,20].

2. Guided shear waves in a plate

The equation representing the free vibrations of a three-dimensional homogeneous and isotropic elastic solid is

$$\mathrm{div}\boldsymbol{\sigma} = \rho\ddot{\mathbf{u}}, \tag{1}$$

where $\ddot{\mathbf{u}}$ is the second order time derivative of the displacement vector, ρ is the material density, $\boldsymbol{\sigma} = \lambda \mathrm{tr}(\mathbf{E})\mathbf{I} + 2\mu\mathbf{E}$ is the stress tensor, with λ and μ the Lamé constants, $\mathbf{I}$ is the identity tensor, and $\mathbf{E} = 1/2(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ is the strain tensor. To solve the problem in plates, we use the partial wave technique, in which wave propagation is treated as a combination of bulk waves reflecting between the boundaries of the plate [27]. We look for plane wavefront (x_2,x_3) solutions propagating in the plane x_1,x_3 (Fig. 1), that is

$$\mathbf{u} = \mathbf{U}e^{i[k(x_1+\alpha x_3)-\omega t]}, \tag{2}$$

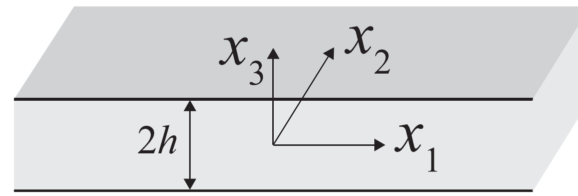

Fig. 1. Plate geometry.

where k is the wavenumber along x_1, α is the ratio of the wavenumber in the x_3 direction to k, and ω is the frequency in rad/s. It is to be noted that in such plane waves, no dependence on x_2 occurs. Substituting Eq. (2) into the equations of motion (1), these decouple into two equations (first and third of (1)) involving displacements along x_1 and x_3, which are the Rayleigh–Lamb waves, and one equation (the second of (1)) involving only displacements along x_2, which represents shear waves. This equation can be written as $\mu(u_{2,11} + u_{2,33}) = \rho\ddot{u}_2$, where the subscripts after the comma denote the spatial derivative. On the basis of the constitutive equation adopted, it must be remarked that, as shear waves involve only displacements along x_2, the stress components are only τ_{23} and τ_{21}. After substitution of (2) into the second of (1), we obtain a first eigenvalue problem, $-\mu k^2(1+\alpha^2) + \rho\omega^2 = 0$, which enables us to determine two values of ratio α, which are $\alpha_{1,2} = \pm\sqrt{c/c_T - 1}$, where $c_T = \sqrt{\mu/\rho}$ is the bulk shear wave velocity and $c = \omega/k$ is the phase velocity of a wave mode with frequency ω and wavenumber k, so that Eq. (2) is given as

$$u_2 = U_2^1 e^{i[k(x_1+\alpha_1 x_3)-\omega t]} + U_2^2 e^{i[k(x_1+\alpha_2 x_3)-\omega t]}. \tag{3}$$

If we set up free-stress boundary conditions at $x_3 = \pm h$, we obtain the second eigenvalue problem as follows:

$$\begin{bmatrix} \mu\alpha_1 e^{ik\alpha_1 h} & \mu\alpha_2 e^{ik\alpha_2 h} \\ \mu\alpha_1 e^{-ik\alpha_1 h} & \mu\alpha_2 e^{-ik\alpha_2 h} \end{bmatrix} \begin{bmatrix} U_2^1 \\ U_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4}$$

which, after some manipulation and considering that $\alpha_1 = -\alpha_2$, provides the characteristic equation $\sin(2k\alpha h) = 0$, whose roots are $\alpha = n\pi/(2kh)$. Substituting in the roots of the first eigenvalue problem, this enables us to determine the phase velocity as a function of ω, as well as the group velocity $c_g = d\omega/dk$ [27].

As an example, the dispersion relation of shear waves showing phase and group velocity as a function of the product $2hf$ in MHz mm, where f is the frequency, is shown in Fig. 2. The case refers to a generic composite fiber-reinforced polymer (FRP) with $\rho = 1600\ \mathrm{kg/m^3}$, $\mu = 68,700$ MPa, height $2h = 3$ mm, and shear bulk wave velocity c_T=6553 m/s. In Fig. 2, continuous curves refer to even symmetric modes and dashed curves to odd nonsymmetric modes. The fundamental zero-order mode has the same velocity for all frequencies and heights, whereas other modes present dispersion. Apart from the fundamental zero mode, every n-th mode has a cutoff frequency equal to $nc_T/(2h)$, below which its wavenumber is imaginary and the wave does not propagate. The mode shapes are derived from Eq. (4), which has to be specialized for even and odd roots, providing respectively $U_2^2 = U_2^1$, and $U_2^2 = -U_2^1$. The related n-th mode shapes are then $U_2(x_3) = U\cos[n\pi x_3/(2h)]$ and $U_2(x_3) = U\sin[n\pi x_3/(2h)]$. Because only displacements according to x_2 are involved in the shear problem, from now on the subscript 2 is dropped. The displacement fields of the first six wave modes are displayed in Fig. 3 for symmetric and nonsymmetric modes. The related components of modal stress are derived from the constitutive relation and depend only on U_2.

3. Interaction of a shear wave with a discontinuity

When a propagating wave encounters a discontinuity of any