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Analytical solution for interaction of an arbitrary frequency curved rectangular inducer with a transverse ring shaped groove surrounding a long conductive cylinder

M.H. Ostovarzadeh *

Department of Electrical and Computer Engineering, Graduate University of Advanced Technology, Haft-bagh Highway, Kerman, Iran

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ABSTRACT

Field distributions around a transverse flaw surrounding a conductive cylinder which is excited by a three-dimensional inducer at arbitrary frequency are presented analytically. The solution is obtained by developing a two-dimensional Fourier series model and using Bessel functions in the third dimension. The metal is assumed as a lossy material and all possible field components in the conductor are expanded. After applying the mode-matching technique, a linear system of AX=B is solved to obtain the unknown coefficients. The accuracy of the proposed modeling technique is confirmed by comparing our results with those obtained by CST finite integration code.

1. Introduction

The Eddy current (EC) [1,2] and alternating current field measurement (ACFM) [3,4] techniques can be used for nondestructive evaluation of metallic structures. These techniques are similar in excitation as an alternating current carrying coil is used to induce eddy currents in test specimen, but are different in measurement and analysis of the resultant magnetic field perturbations caused by a flaw, because in EC technique, the change of impedance of exciting coil is measured to predict the metal condition, but in ACFM technique the change of surface magnetic fields measured by a sensor is used to disclose the metal condition.

The theoretical prediction of EC and ACFM probes output signals (*i.e.*, the "forward problem") involves the solution of electromagnetic field for a current-carrying inducer in the vicinity of a flawed metal. When compared with numerical solution methods, analytical methods, which use eigenfunction expansion [5] to solve the governing equations, are often more appealing for the generality of the solution in the form of closed mathematical relations and giving more insight to the eddy current distributions and relatively fewer computation resources. These features plays an important role in solving the so-called "inverse problem" (particularly in real-time applications) where the unknown geometry of a crack is to be determined iteratively, by repetitive calculation of probe output signal for a given crack geometry [6] or by constructing database for training an artificial neural network [7].

In EC testing, the depth of eddy current can be increased by lowering the exciting frequency. Therefore, inspection can be performed at various depths of the test specimen in a single scan by using multi-frequency/pulsed excitation [8–10].

Analytical solutions for arbitrary frequency excitation are available for flawless metallic slabs [11,12], cylinders [13–17]. In the case of flawed work pieces, Bowler and Theodoulidis studied the effect of a conductive wedge on the impedance of a 3D inducer [18]. They also investigated the effect of a long crack in a metallic slab excited by a two dimensional (2-D) inducer [19] at arbitrary frequency. In recent works, analytical solutions were proposed for field distributions around surface and hidden cracks in conductive half spaces excited by 3D inducers [20–22]. In these works, the metal was assumed as a lossy dielectric and a new TM potential representing the effect of displacement current through the narrow opening of the crack was introduced.

In this paper, the works is extended to the problem of field distribution due to a 3D inducer located around a conductive cylinder including a surface ring shaped groove. This problem describes an analysis to account for the presence of approximately long cracks located transversely around conducting rods.

The paper is organized as follows. In Section 2, we present the problem and its formulation where the problem is divided in two even- and odd-symmetry problems. The solutions to the resultant

* Tel.: +98 343 622 6611 13; fax: +98 343 622 6617.
E-mail address: mh.ostovarzadeh@kgut.ac.ir

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even and odd problems are described in Sections 3 and 4, respectively. In Section 5, the results of the proposed method for field distribution due to a 3D inducer are compared with those obtained using a commercial finite integration code.

2. Problem formulation

The test specimen is a conductive cylinder of radius a with a constant conductivity σ, constant permeability μ_0 and constant permittivity ε (Fig. 1). From it, a ring-shaped groove of depth d and width g has been carved. The groove consists of two parallel faces lying on the r–φ plane, perpendicular to the axis of the test specimen (z-direction) and separated by a distance g in the z-direction. The surface of the test specimen is interrogated by the field of a curved rectangular current-carrying wire located at a fixed radius h. Because of existing symmetry of the problem in the φ- direction, the curvature of inducer is assumed to be between angles $-\varphi_0$ and φ_0. The inducer carries an alternating current of arbitrary frequency f and magnitude I.

To solve the problem posed above, we recognize three regions, namely, region I (the air, outside the cylinder, $r \geq a$), region II (in the flawed part of the cylinder, $a-d \leq r < a$) and region III (inside the cylinder, below the groove, $r < a-d$).

In region I, the divergence-free magnetic field can be obtained from the Hertz potential W (*i.e.*, $\vec{H} = \nabla(\partial W/\partial z)$ [15,17]. To simplify the solution process, we express W in terms of two components with the Laplacian distributions as follows:

$$W = W_\text{i} + W_\text{p} \tag{1}$$

where W_i and W_p are, respectively, the potential functions representing the incident field in the absence of the metal and the field perturbation due to the metal and the groove.

The solution of incident field in the region underneath the curved inducer can be expressed as follows [15]:

$$W_\text{i}(r,\varphi,z) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C^\text{i}(\alpha,m) \text{I}_m(|\alpha|r) e^{jm\varphi} e^{j\alpha z} d\alpha \tag{2}$$

where I_m denotes the modified Bessel function of the first type and m and α are, respectively, the Fourier variables associated with the φ- and z- directions. The expression for source coefficient $C^\text{i}(\alpha,m)$ due to a curved rectangular inducer with dimensions shown in Fig. 1 is obtained as follows [15].

$$C^i(\alpha,m) = \frac{Ih}{2\pi^2} \text{K}'_m(|\alpha|h) \frac{e^{-j\alpha z_1} - e^{-j\alpha z_2}}{|\alpha|} \frac{\sin(m\varphi_0)}{m} \tag{3}$$

where K_m denotes the modified Bessel function of second type, h is the radius of the curved inducer, φ_0 is half of the angle of the inducer curvature in the φ- direction and z_i are the coordinates of the corners of the inducer in the z-direction.

In region II, the problem extends to air (groove) and the metal. To obtain the solution in this region, we assume the metal as a lossy material with dielectric constant $\varepsilon^* = \varepsilon - j\sigma/\omega$ where $\omega = 2\pi f$ and use the magnetic potential vector, $\vec{A} = A_z\hat{z}$ and the electric potential vector $\vec{F} = F_z\hat{z}$ to expand, respectively, TM_z and TE_z modes in this region. To complete the solution in region II, we also need to include the effect of eddy current by introducing an extra potential function, $\vec{A}' = A'_z\hat{z}$ [18].

Using the Maxwell equations, one can arrive at the following expressions for $\vec{A}$:

$$\nabla^2\vec{A} + \omega^2\mu_0\varepsilon_0\vec{A} = 0 \quad \text{in air} \tag{4 – a}$$

$$\nabla^2\vec{A} = j\omega\mu_0(\sigma + j\omega\varepsilon)\vec{A} \quad \text{in metal} \tag{4 – b}$$

where μ_0 and ε_0 are, respectively, the air permeability and dielectric constant. The corresponding magnetic field $\vec{H}$ and electric field $\vec{E}$ are as follows:

$$\vec{H} = \frac{1}{\mu_0}\nabla \times \vec{A} \tag{5}$$

$$\vec{E} = -j\omega\vec{A} + \frac{1}{j\omega\mu_0\varepsilon}\nabla\nabla.\vec{A} \tag{6}$$

where ε is the dielectric constant of the corresponding region.

Similarly, one can derive the following expressions for $\vec{F}$:

$$\nabla^2\vec{F} + \omega^2\mu_0\varepsilon_0\vec{F} = 0 \quad \text{in air} \tag{7 – a}$$

$$\nabla^2\vec{F} = j\omega\mu_0(\sigma + j\omega\varepsilon)\vec{F} \quad \text{in metal} \tag{7 – b}$$

the fields dependencies to this potential are:

$$\vec{E} = -\frac{1}{\varepsilon}\nabla \times \vec{F} \tag{8}$$

$$\vec{H} = -j\omega\vec{F} + \frac{1}{j\omega\mu_0\varepsilon}\nabla\nabla.\vec{F} \tag{9}$$

In region III, we have a lossy dielectric (metal). To model all possible field distributions, two potential functions $\vec{A} = A_z\hat{z}$ and $\vec{F} = F_z\hat{z}$ which, respectively, satisfy in Eqs. (4-b) and (7-b) are introduced. The field dependencies to these potentials are as (5), (6) and (8), (9), respectively.

The final solution of the problem is obtained by ensuring that the boundary conditions between the three regions are satisfied.

For simplicity, we split the solution into two even and odd solutions with respect to the z- direction [19–22] which are described in the following sections.

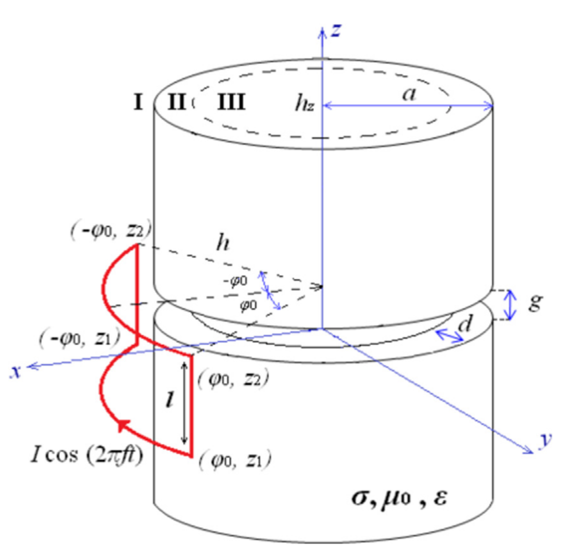

Fig. 1. A curved rectangular inducer above a conductive cylinder with a transverse groove.

3. Even symmetry solution

As described in [15], in region I, we have a potential function W with the Laplacian distribution. By truncating the solution at a large distance from the inducer such as h_z in the z-direction, expression for the even component of W in region I is as follows