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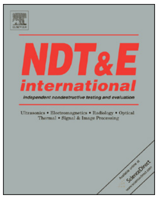

Time-domain analytical solutions to pulsed eddy current field excited by a probe coil outside a conducting ferromagnetic pipe

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ABSTRACT

Using the second-order vector potential formalism and block matrix, the non-axisymmetric eddy current field induced by a probe coil positioned perpendicularly outside a conducting ferromagnetic pipe is solved analytically. Then, the time-domain expressions of induced voltage and eddy current density in the pipe are obtained through the Laplace inverse transformation, which is carried out by calculating the residues of poles. Furthermore, the diffusion process of pulsed eddy current in the pipe is examined. Finally, the analytical solutions are verified through the experiment results of two steel pipes with different wall thickness.

1. Introduction

In oil and gas, chemical, electric power, metallurgy and other related industries, ferromagnetic metallic pipes are widely used to transport liquid or gaseous corrosive media, and most pipes usually work under conditions of high temperature and high pressure. As a result, corrosion of ferromagnetic metallic pipes is a kind of unavoidable and widespread damage. Wall-thinning defects usually caused by corrosion are potential hazards to the safety, and may lead to pipeline leakage, explosion or other accidents. Therefore, to ensure safe operation, regular in-service nondestructive testing and evaluation for remaining wall thickness of ferromagnetic pipes are indispensable.

Eddy current testing (ECT) is a non-contact electromagnetic inspection method which can be used through the insulation and cladding. However, the conventional ECT method excited by time-harmonic current has rarely been used to measure the wall thickness of ferromagnetic components due to the limitation of its penetration depth. Pulsed eddy current testing (PECT) is an alternative method which has been proved to be effective for inspecting coated ferromagnetic pipes and containers [1–3]. Instead of time-harmonic excitation current, a pulsed current is used to induce a pulsed magnetic field outside the conducting ferromagnetic pipe, and then the wall-thinning corrosion of the pipe can be evaluated by detecting this pulsed eddy current electromagnetic field.

Analytical solutions to axially symmetric eddy current problems have been developed perfectly. By the separation of variables, one classical method is proposed in [4,5] to achieve the closed-form expressions of the magnetic vector potentials for an infinitely long, conducting rod or pipe encircled with a coil. However, with non-axisymmetric models, the method in [4] is no longer appropriate to solve the eddy current field. Fortunately, the second-order vector potential (SOVP) has been proved to be an effective tool to solve non-axisymmetric problems, such as a conducting cylinder placed in an eccentric circular current loop [6], an eccentric coil inside a conducting pipe [7], and a bobbin coil in a conducting pipe with eccentric walls [8]. Meanwhile, the closed-form expressions of the impedance change for a coil positioned outside a conducting cylinder are solved by the SOVP formulation [9,10]. Furthermore, the model is extended to a conducting pipe and solved analytically through the similar method [11–13].

On the basis of these analytical solutions to time-harmonic eddy current field, time-domain expressions of pulsed eddy current field can be calculated through the Laplace inverse transformation. By means of the residue theorem, the time-domain analytical solutions to the pulsed magnetic field for enclosures and planar slabs are obtained through the Laplace inverse transformation, which is carried out by calculating the residues of poles [14,15]. Based on this method, [16,17] propose an approach to solve the time-domain solutions to the pulsed eddy current field for a conducting plate using the Heaviside expansion

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theorem, and the time-domain expressions of the induced voltage in the form of series are obtained.

The theoretical model of pipes were usually simplified to a plate model in the PECT [2,3], unfortunately, great error of the model has been caused to study the pulsed eddy current field of pipes. On the basis of the analytical solutions to the time-harmonic eddy current field of pipes obtained by us in previous work, the purpose of this paper is to solve the time-domain analytical expressions of the voltage across the pick-up coil and the eddy current density in the pipe wall, which are induced by a pulsed current in the drive coil positioned perpendicularly outside a conducting ferromagnetic pipe. This paper is organized as follows. In Sections 2 and 3, the frequency-domain and time-domain solutions to the eddy current field for a conducting ferromagnetic pipe are solved respectively. Next, the diffusion process of the pulsed eddy current in the pipe wall is examined in Section 4. Then, theoretical calculated results are compared with experimental results in Section 5. Finally, the conclusions are given in Section 6.

2. Frequency-domain solutions to eddy current field for a conducting ferromagnetic pipe

2.1. The ECT model

The ECT model of an infinitely long, conducting, ferromagnetic pipe is shown in Fig. 1. Respectively, r_1 and r_2 are the pipe's inner radius and outer radius; d is the wall thickness, with $d = r_2 - r_1$; σ and μ are the ferromagnetic pipe's conductivity and permeability, where $\mu = \mu_0 \mu_r$ (μ_0 is the permeability of the vacuum, μ_r is the relative permeability). An air-cored cylindrical drive coil (subscript d) and pick-up coil (subscript p) of height h are positioned outside the pipe with their central axis perpendicular to the pipe axis and coincident with the pipe radius, as shown in Fig. 1. l_o is the lift-off between the probe coils and the outer surface of the pipe. The geometric parameters of the probe coil assigned in this paper are listed in Table 1. A cylindrical coordinate system $O\rho\varphi z$ is established with the z-axis coincident with the pipe axis. As shown in Fig. 1(b), the field region is divided into 3 sub-regions: region 1, $\rho < r_1$; region 2, $r_1 < \rho < r_2$; region 3, $r_2 < \rho < r_2 + l_o$.

2.2. Frequency-domain solutions to the eddy current field

Ignoring the effects of displacement current in Fig. 1, the magnetic vector potential $\boldsymbol{A}$ satisfies the vector Helmholtz equation in frequency-domain [5]

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = 0 \tag{1}$$

where, $k^2 = 0$, in non-conducting regions 1 and 3; $k^2 = -\mu\sigma s$, in conducting region 2. It is difficult to solve Eq. (1) directly for the 3D time-harmonic eddy current field presented in Fig. 1. Fortunately, this non-axisymmetric model can be solved by the SOVP formulation $\boldsymbol{W}$, which is defined as

$$\boldsymbol{A} = \nabla \times \boldsymbol{W} = \nabla \times (W_a \boldsymbol{e}_z + \boldsymbol{e}_z \times \nabla W_b) \tag{2}$$

where $\boldsymbol{e}_z$ is the unit vector in z direction, W_a and W_b are second-order potential functions. It has been proved that in non-conducting region, the magnetic flux density can be described using W_a only, and the knowledge of W_b is not required [18]. From Eqs. (1) and (2), it is deduced that W_a and W_b satisfy the scalar Helmholtz or scalar Laplace equations in conducting and non-conducting regions respectively

$$\nabla^2 W_a + k^2 W_a = 0 \tag{3}$$

$$\nabla^2 W_b + k^2 W_b = 0 \tag{4}$$

using the separation of variables method in cylindrical coordinate, applying physical restrictions for the potentials at $\rho \to 0$, and considering the symmetries of the magnetic field about φ and z

Table 1
Parameters of the drive coil and pick-up coil.

Parameters	Drive coil	Pick-up coil
Number of turns N	174	1025
Height h (mm)	25.0	25.0
Inner radius r_i (mm)	20.9	20.0
Outer radius r_o (mm)	22.2	20.8

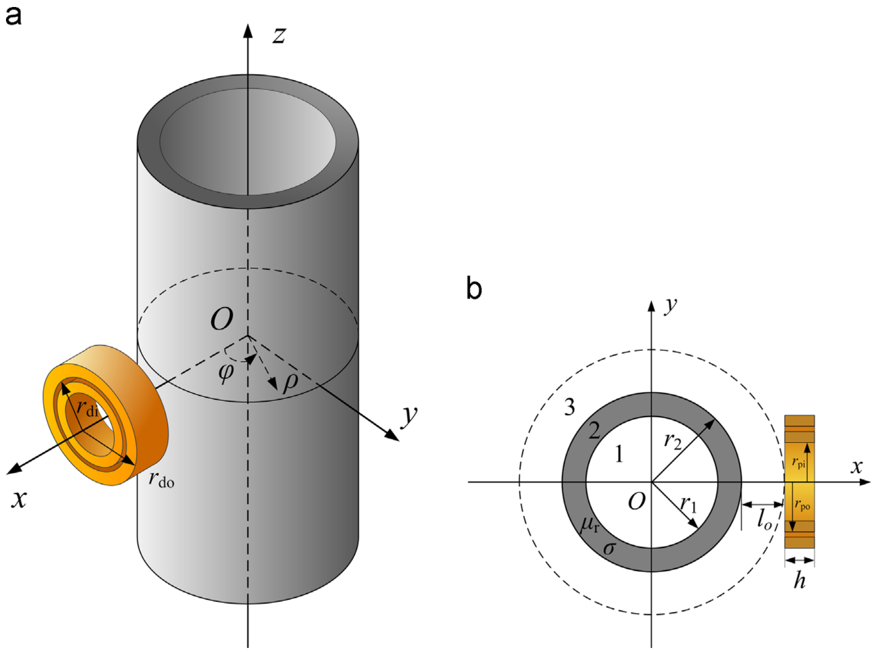

Fig. 1. (a) 3D diagram and (b) top view of the ECT model of two air-cored cylindrical coils positioned perpendicularly outside a conducting ferromagnetic pipe.