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Evaluation of dispersion characteristics of multimodal guided waves using slant stack transform

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ABSTRACT

In this paper slant stack (*SL*) transform is presented and its application for processing of multi-modal dispersive Lamb waves snapshots is proposed. The *SL* transform can facilitate the evaluation of dispersion curves based on a set of signals captured at the structure's surface. The presented technique leads explicitly to the frequency-phase-velocity representation of the processed signals. Theory behind the technique is presented and the *SL* results are compared to those obtained using the 2D discrete Fourier transform. The *SL* is used to process data acquired from an aluminum plate and to investigate anisotropic properties of a composite plate.

1. Introduction

In the recent years Lamb waves have shown a great potential in ultrasonic nondestructive testing (NDT) applications [1]. The ability of Lamb waves to travel over long distances and their sensitivity to different types of flaws is the advantages that enable inspection of large planar structures. However, a serious issue to deal with in such applications is the multimodal and dispersive nature of Lamb waves. Multiple modes can simultaneously propagate in a structure with different velocities and therefore, even for an intact structure, the acquired ultrasonic signals normally include a number of superimposed wave-packets corresponding to the excited modes, which complicates the process of extracting damage-reflected waves. Additionally, dispersion effects are often apparent, i.e., different frequency components of a particular propagating mode travel with different velocities, which results in the signal elongation in time and reduces range resolution of inspection systems [2].

Therefore, in many applications, precise information concerning both modes' content and wave velocity in the investigated structure is often assumed. For instance, in phased array imaging algorithms wave velocity is needed for beam-steering and for the evaluation of the distance from scatterer to the array [3]. Local change of wave velocity observed in the inspected structure can be used as an important damage symptom, e.g., such changes can be observed due to the mode-conversion at a damage interface or due to the thickness changes in delaminated area [4,5].

Lamb wave velocity can be predicted theoretically [6], however, precise information concerning material properties and plate thickness is required for that. Also the inverse problem can be formulated – measurements of the dispersive characteristics together with predictions generated by a suitable theoretical model can be used for the elastic constants' estimation [6].

Numerous methods for wave velocity measurement have been proposed. In the case of non-dispersive bulk waves, the knowledge of propagation distance and time of flight (TOF) measurement is sufficient for the velocity evaluation. The TOF can be captured as a time interval between the maximum peaks in the excitation pulse and the response. In the case of dispersive waves, this method can be, to some extent, used for group velocity measurements. The TOF is considered then as a time lag between peaks in the envelopes of excitation and response [7]. Since in the case of dispersive waves, different frequency components travel with different velocities, this method allows only for the assessment of group velocity, i.e., the velocity of energy propagation.

For the estimation of phase velocity precise time and frequency information concerning the analyzed signal is needed. This information can be acquired by gathering data either for several frequencies or for several points in space. A continuous sine wave excitation signal can be used to measure the TOF of a single precisely determined frequency component as it was reported in [6]. The measurement was performed by varying the separation in a transmitter/receiver pair to obtain

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phase-matching between the excitation and the response. Although this technique can be accurate for a single mode and a single frequency, its application for the evaluation of dispersion curves for a broad-band frequency range seems to be both impractical and time-consuming.

Another problem encountered when measuring dispersion characteristics is the existence of more than one mode for a single excitation frequency. Moreover, in NDT applications not only the value of the phase velocity but also the amplitude of the wave-mode at a given frequency range is important. This information can be used to select the frequency range in which a considerable suppression of undesired modes can be obtained. Therefore, spectral methods, which enable to evaluate amplitude and velocity of different modes are an interesting tool for the measurement of dispersion curves. An example of such a technique, applied to ultrasonic guided waves is the 2D discrete Fourier transform (2DDFT) [8]. This technique involves a broadband excitation signal and a set of signals captured in subsequent points along the wave propagation path. The acquired data is then processed using the 2DDFT that yields the frequency–wavenumber representation of the dispersive waves.

In this paper an alternative method for the experimental evaluation of dispersion characteristics is presented. The principle of this method is similar to that presented by Kebaili and Schmitt [9] who evaluated phase velocities in laminates with orthorhombic symmetry over a broad range of propagation angles. The technique has its origin in geophysics where spectral analysis of surface waves (SASW) is often used for ground structure recovery. A conventional SASW method, which in principle uses two sensors, is often ineffective for noisy signals and has recently been replaced by the multichannel analysis of surface waves (MASW) [10]. For these reasons, the MASW is arguably regarded as a better approach than the conventional SASW. When the MASW setup is applied, the estimation of dispersion characteristics of surface wave field is commonly determined using the so-called p-τ method [11] or the slant stack (*SL*) transformation [12,13].

The *SL* method, which is a digital form of Radon transform, is a widely used technique for analyzing high-quality reflection and refraction data in geophysics. The transform is applicable to data from a line source in a plane model, that is, one Cartesian coordinate. The *SL* essentially decomposes the locus of the directly arriving waveforms into the equivalent set of elementary plane waves according to their direction of propagation.

Recently, it has been applied to estimate the dispersion characteristics of dispersive Rayleigh waves in application for NDT of concrete structures [14,15]. This technique can be also easily introduced for the evaluation of guided waves, e.g., Lamb waves in various applications of ultrasound. Therefore, a detailed presentation of this method, which is probably unfamiliar to the majority of the readers, and its application for processing signals captured in plate-like structures will be presented in this paper. Both numerical and experimental data will be used to illustrate the *SL* method's performance.

This paper is organized as follows. First, a brief introduction to Lamb waves and their dispersive and multimodal nature will be given in Section 2.1. Next, the theoretical background of the *SL* transform will be presented in Section 2.2. Performance of the method and its relation to 2DDFT will be discussed using simulated signals in Section 3. Finally, the *SL* transform will be used for the evaluation of dispersion curves of an aluminum and a composite plate. The experimental results will be presented in Section 4, followed by conclusions given in Section 5.

2. Theoretical background

We start our presentation with a brief description explaining how dispersion influences Lamb wave signals. Next, the principles of *SL* transform will be outlined, followed by the comparison of the presented technique with the 2DDFT method.

2.1. Phase velocity and dispersion

Let us first assume a plane, harmonic wave with amplitude A, angular frequency ω and wavenumber k, propagating in non-attenuating and non-dispersive medium. The displacement resulting from the wave propagation can be described using a general analytical expression

$$y(t,x) = A \cdot \cos(\omega t - kx) \tag{1}$$

Velocity of this wave is related to the phase difference between the vibrations measured at two different points during the propagation of the wave. Rewriting the above equation in the form

$$y(t,x) = A \cdot \cos \cdot \omega\left(t - \frac{k}{\omega}x\right), \tag{2}$$

reveals wave phase velocity defined by the argument of the cosine function as $V_{ph} = \omega/k$, [16]. For an elementary case of non-dispersive medium the group V_{gr} and phase velocities are equal, i.e., $V_{gr} = V_{ph} = const$ [16], and the analytic form of the time-domain wide-band plane-wave signal $y(t,x)$ received at the distance x can be easily predicted as

$$y(t,x) = s(t) \circledast \delta\left(t - \frac{x}{V_{ph}}\right) = s\left(t - \frac{x}{V_{ph}}\right), \tag{3}$$

where $s(t)$ is the excitation displacement signal. In the frequency domain Eq. (3) takes the following form:

$$Y(\omega,x) = S(\omega) \cdot e^{-j(\omega/V_{ph})x}. \tag{4}$$

If the phase velocity of a wave traveling in a medium is a function of frequency the medium is referred to as *dispersive*. The dispersive nature of a medium can be related to its various features, for instance, geometric dispersion (related to the presence of specimen boundaries), material dispersion (present when material's elastic constants depend on frequency) or scattering dispersion (caused by the presence of densely distributed inhomogeneities in the material) [17].

Lamb waves that are formed by a superposition of multiple reflections and mode conversions of longitudinal and shear waves at the free surfaces of the plate generally exhibit strong geometric dispersion [6]. Harmonic guided waves, propagating in the plane of the plate, may exist only for those combinations of frequency and phase velocity corresponding to the existence of standing waves in the thickness direction. The set of permissible frequencies can be obtained by solving the Rayleigh–Lamb equations. The resulting phase velocities plotted as a function frequency-thickness product are referred to as the dispersion curves [16]. An example of the dispersion curves calculated for an isotropic aluminum plate is shown in Fig. 1. The curves correspond to the successive antisymmetric (flexural) and symmetric (longitudinal) modes that can propagate for each frequency-thickness value.

In the case of dispersive medium the phase velocity becomes a function of frequency and Eq. (4) takes the form

$$Y(\omega,x) = S(\omega)e^{-j(\omega/V_{ph}(\omega))x}. \tag{5}$$

Note the signal's amplitude can decrease with the propagation distance due to material attenuation, energy leakage and geometrical spreading of the wavefront. These changes are, however, not mentioned in the equations for the sake of clarity.

In theory, phase velocity can be evaluated by SASW from the phase $\phi(\omega,x) = (\omega/V_{ph}(\omega))x$ of the signal $Y(\omega,x)$ using the two-point measurement of the phase difference $\Delta\varphi$ between the points spaced at a distance d. This method, however, is impractical for Lamb waves since, as shown in Fig. 1, multiple modes may exist simultaneously and therefore for higher frequency bands' phase velocity is not unique. Indeed, taking into account the multimodal nature of Lamb waves and assuming that q modes are excited in a