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Concerning the derivation of exact solutions to inductive circuit problems for eddy current testing

D.R. Desjardins [a,*], T.W. Krause [b], A. Tetervak [a,b], L. Clapham [a]

[a] Department of Physics, Queen's University, Kingston, Ont, Canada, K7L 3N6
[b] Department of Physics, Royal Military College of Canada, Kingston, Ont, Canada, K7K 7B4

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ABSTRACT

A novel strategy, which enables the derivation of exact solutions describing all electromagnetic interactions arising in inductively coupled circuits, is developed. Differential circuit equations are formulated in terms of the magnetic fields arising in inductive systems, using Faraday's law and convolution, and solved using the Fourier transform. The approach is valid for systems containing any number of driving and receiving coils, and may be extended to include nearby conducting and ferromagnetic structures. Furthermore, arbitrary excitation waveforms, such as a sinusoid or a square wave for applications in conventional and transient eddy current, respectively, may be considered. In this first work, the general theory is presented and subsequently applied to the simple case of a coaxial driver and receiver coil configuration. Theoretical expressions for the self- and mutual inductance coefficients are shown to fall out of the theory naturally. Experimental results, obtained for a square wave function excitation, are found to be in excellent agreement with the analytical predictions.

1. Introduction

Eddy current non-destructive testing (NDT) is one of several technologies making use of the precise generation and measurement of magnetic fields. Mathematical models describing the transient electromagnetic interactions, which arise between field generating and sensing coils in close proximity to conducting structures, enable the determination of these structures' geometrical and material characteristics. This capability is critically important to the aeronautical, nuclear and petrochemical industries for example [1], which monitor the structural health of their physical assets for economic and safety reasons. Research in eddy current theory had already begun by 1921 when Von Wwedensky [2] calculated the diffusion of magnetic fields impressed upon rigid conducting cylinders. Later, the pioneering work of Dodd and Deeds [3] in the late 1960s had led to the formulation of solutions to harmonic induction problems using a magnetic vector potential formalism. More recently, theoretical work has been performed by Morozova [4], Fan [5], Bowler [6–8], Theodoulidis [9,10] and many others [11–14] with the aim of modeling induced voltages or coil impedance changes for applications in NDT. A persistent challenge to the development of driver-pickup models, however, has been a lack of agreement with experimental results in cases where magnetic structures, which exhibit strong feedback effects, are considered. This is quite inopportune since steel, and many other materials commonly encountered in industry, are ferromagnetic. Furthermore, the additional magnetic flux, generated by the magnetization of such structures, enhances the induced eddy current densities [15]. These incentives provide the motivation for the development of exact mathematical models, which would facilitate the quantitative analysis and interpretation of experimental signals.

This work is the first to incorporate electromagnetic field solutions directly into Kirchoff's circuit equations. The strategy yields exact expressions that describe all the electromagnetic interactions arising in inductively coupled systems. Thus, feedback effects, which have historically posed a challenge to the development of transient eddy current models, are completely addressed. In this work, the methodology for solutions to general eddy current problems is developed in full, but only applied to the simpler case of a coaxial driver and pickup coil configuration. Initial validation of the theory in this simple case demonstrates the effectiveness of the methodology. In subsequent works, the theory will be applied to inductive circuit problems containing ferromagnetic conducting structures, such as half-spaces, plates, rods and tubes. Thus, in a series of works that will follow, the complexity and number of variables considered will be iteratively increased along with more general validation of the theory and exploration of its potential applications.

* Corresponding author.
E-mail address: dandesja@gmail.com (D.R. Desjardins).

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In the simpler case, where no conducting structure is present, electromagnetic interactions arise solely from the inductive coupling of the coils. To this end, solutions to two-coil inductive circuit problems have already been obtained [16] in the traditional manner; differential circuit equations are formulated in terms of constant coefficients L and M, the self- and mutual inductance coefficients, respectively. However, the manner in which L and M come to appear in these equations, although understood, is not explicitly shown. In what follows, the circuit equations are formulated from first principles using Faraday's law and convolution theory [17]. In doing so, the correct solutions, which describe the transient currents flowing in the coupled driver and pickup circuits, are achieved and validated experimentally. More importantly, expressions for L and M are shown to fall out of the theory naturally. This work is in preparation for subsequent addition of other inductive elements. In particular, it will be shown in future work that additional inductance effects, associated with the presence of ferromagnetic conducting structures, emerge from the theory.

2. Theoretical development

In consideration of what follows, a circular pickup coil is centered about the axis of a larger encircling driver coil. In accordance with Maxwell's equations, a time-varying current flowing in the driver coil will induce a current in the pickup. The current induced in the pickup gives rise to a transient magnetic field, which, in turn, generates an opposing current within the driver coil. The circuit equations describing the resultant time-dependent currents i_1 and i_2 flowing in the driver and pickup coils, respectively, are written using Kirchhoff's laws in the following general form:

$$R_1 i_1(t) = v(t) + \varepsilon_1(t) \quad (1)$$

$$R_2 i_2(t) = \varepsilon_2(t)\,, \quad (2)$$

where v is any time-dependent excitation voltage – step, harmonic, multi-frequency, ramp, saw-tooth, etc. – R_1 and R_2 are the total circuit resistances, and ε_1 and ε_2 are the total time-dependent voltages induced in the driver and pickup coils, respectively. Both ε_1 and ε_2 have three components; one arising from the field generated by the driver coil, another from the field generated by the receiver coil and the third from transient eddy current fields emanating from a conducting sample when present. A challenge in the development of analytical transient eddy current models has been the inability to formulate expressions for these feedback terms.

In what follows, induced voltages ε_1 and ε_2 are expressed in terms of the unknown current functions i_1 and i_2 using convolution integrals, and substituted back into the circuit equations. Beginning with Faraday's law, a voltage ε is induced within a current loop C, and is proportional to the time-derivative of the magnetic flux normal to the area, S, enclosed by that loop such that [18]

$$\varepsilon(t) = -\frac{\mathrm{d}}{\mathrm{d}t}\oiint_S \mathbf{B}(r,z,t)\cdot \mathrm{d}\mathbf{S}\,, \quad (3)$$

where d$\mathbf{S}$ is the differential surface element. A magnetic vector potential formalism is adopted in order to take advantage of the circular (azimuthal) symmetry of the problem. The magnetic flux density $\mathbf{B}$ is written as the curl of $\mathbf{A}$, and in accordance with the Stokes' theorem, the surface integral is recast into a contour integral. Finally, $\mathbf{A}$ contains only an azimuthal vector component and is symmetric about the axis. Thus, the voltage induced within a current loop due to a time-varying vector potential is expressed as

$$\varepsilon(t) = -\frac{\mathrm{d}}{\mathrm{d}t}\oiint_S (\nabla\times\mathbf{A}(r,z,t))\cdot \mathrm{d}\mathbf{S} = -\frac{\mathrm{d}}{\mathrm{d}t}\oint_C \mathbf{A}(r,z,t)\cdot \mathrm{d}\mathbf{l} = -2\pi r\frac{\mathrm{d}}{\mathrm{d}t}A(r,z,t)\,. \quad (4)$$

For a circular multi-turn coil with a rectangular cross-section, the current loop expression in Eq. (4) is integrated over the cross-sectional area of the coil Σ, and multiplied by a turn density so that

$$\varepsilon(t) = -\frac{2\pi N}{l(b-a)}\frac{\mathrm{d}}{\mathrm{d}t}\oiint_\Sigma rA(r,z,t)\,\mathrm{d}r\,\mathrm{d}z\,, \quad (5)$$

where N is the number of turns, l is the coil length, a and b are the inner and outer coil radii, respectively. The time-dependence of the magnetic vector potential A is not explicitly known. It can, however, be expressed as the convolution of the system's unit impulse response [17] with a yet unknown time-dependent current function such that

$$A(r,z,t) = \hat{A}(r,z,t) * i(t) \equiv \int_0^\tau \hat{A}(r,z,t-\tau)i(\tau)d\tau\,, \quad (6)$$

where $\hat{A}$ is the system's unit impulse response and A is the actual system response resulting from the onset of the time-dependent current i. Thus, A inherits the time-dependence and relative magnitude of i, as expected. In an inductive system composed of m coils, there are m unit impulse response solutions, associated with their respective current functions, existing in superposition. The straightforward generalization of Eq. (6), which describes the total magnetic vector potential A in a multi-coil system, is

$$A(r,z,t) = \hat{A}_1(r,z,t) * i_1(t) + \hat{A}_2(r,z,t) * i_2(t) + \ldots + \hat{A}_m(r,z,t) * i_m(t)\,, \quad (7)$$

where the number in the subscript angle-brackets ⟨⟩ specifies the coil from which the potential arises. Eq. (7) is substituted into (5) and the total time-dependent voltage induced in the kth coil of a system of m coils, now expressed in terms of the unknown current functions, may be written as

$$\varepsilon_k(t) = -\frac{2\pi N_k}{l_k(b_k-a_k)}\frac{\mathrm{d}}{\mathrm{d}t}\oiint^{[k]} r\left(\hat{A}_{\langle 1\rangle} * i_1 + \hat{A}_{\langle 2\rangle} * i_2 + \cdots + \hat{A}_{\langle m\rangle} * i_m\right)\mathrm{d}r\mathrm{d}z\,, \quad (8)$$