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A theoretical model of fluidelastic instability in tube arrays

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ABSTRACT

A theoretical model of the fluidelastic instability in tube arrays is presented in this article. It is developed for a normal-square cylinder array and then extended to other types of array patterns. The model is based on transient interactions between a single cylinder and the adjacent flow streams of single phase fluid. The central cylinder is assumed to oscillate as a one-degree-of-freedom mass on a spring system in the lift direction only. A small displacement of cylinder is assumed to perturb the surrounding interstitial flow, while as for higher displacements the cylinder causes flow distortions in regular intervals. These disturbances are convected downstream along with the interstitial flow. The waveforms of these flow distortions are assumed to interact with the array pattern, thence modifying the fluid force acting on the cylinder. The critical flow velocity is obtained as a function of mass ratio and damping parameter. The proportionality constant of the mathematical model is derived in terms of the pitch ratio and Euler number. The mathematical development results in an implicit model for the critical flow velocity. The model predictions are in a good agreement with experimental results.

1. Introduction

The flow-induced vibrations in the heat exchanger tube arrays exhibit different mechanisms. The vibrations are generally classified under, vortex-induced vibration, turbulent buffeting, acoustic vibration and the fluidelastic vibration. The underlying mechanisms in the first three types of vibration are well understood. The safe operating conditions can be procured against these vibration types by appropriate design guidelines. The exact mechanism underlying the fluidelastic instability is relatively less understood. The damage due to the fluidelastic instability is generally severe and occurs within relatively short time. The fluidelastic instability is extensively studied in order to accurately understand and predict the critical velocity thresholds. The presence of fluidelastic excitations in the context of cylinders was first reported in Roberts (1962). The work of Connors (1970) and Connors, 1978 led to a simplified model for the fluidelastic instability,

$$\frac{u_{pc}}{f_n D} = K\left(\frac{m\delta}{\rho D^2}\right)^a \tag{1}$$

where, u_{pc}, f_n and D are the critical pitch (minimum gap) velocity, natural frequency and the diameter of the cylinder respectively. The non-dimensional critical pitch velocity is proportional to the mass m, logarithmic decrement δ of the cylinder vibration in the non-dimension forms with the exponent a. K is the constant of proportionality. ρ is the fluid density. An enormous amount of work is carried out in terms of experiments and theoretical models, since the work of Connors (1970), in order to better understand and predict the phenomenon. The topic is well reviewed in Païdoussis (1983), Weaver and Fitzpatrick (1988), Pettigrew and Taylor (1991) and more recently in Païdoussis et al. (2010, Chapter 5). A detailed review on the mathematical models of fluidelastic instability is provided in Price (1995).

In this article, a new mathematical model for the fluidelastic instability is presented. It is based on dynamic interactions between a single cylinder and its adjacent fluid flow. The flow perturbations due to the cylinder motion are modeled as waveforms on top of the interstitial fluid flow. The flow streams carrying these perturbations interact elastically with the cylinder, especially for the low mass ratio ($m/\rho D^2$). The mathematical development and a procedure to estimate the critical pitch velocity u_{pc} is formulated in the following sections. The model predictions are compared with a set of experimental data listed in Pettigrew and Taylor (1991) as well as with a large experimental data reported in Païdoussis et al. (2010, Chapter 5)) for all the four array patterns.

2. Theory

The cross flow in normal-square tube arrays forms a typical flow pattern, which consists flow channels with varying cross-sectional area.

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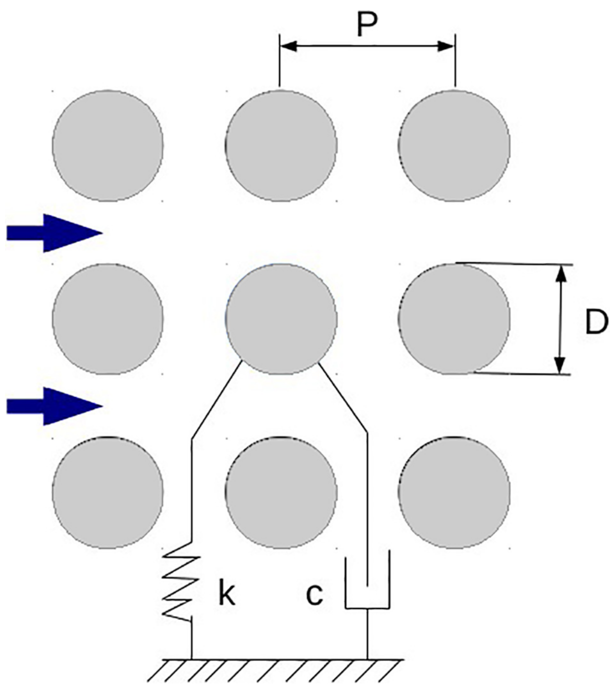

Fig. 1. The kernel of a normal-square cylinder array.

The interstitial flow velocity varies depending on the cross-sectional area. The flow accelerates between adjacent cylinders of a row (lower cross-sectional area), while as it decelerates between the two rows of cylinders (larger cross-sectional area). A motion of cylinder in the flow normal (or lift) direction results in a decrease in the cross-sectional area of an adjacent flow channel on one side of the cylinder, and at the same time an increase in the cross-sectional area on the other side of the cylinder. Consequently, the local flow velocity either increases or decreases accordingly. These perturbations are conveyed away from the cylinder, mainly, in the downstream direction along the flow. These perturbed flow channels dynamically interacts with the cylinder. The mathematical model proposed in the following section is based on these dynamic interactions between the flow streams and a cylinder of the normal-square (90°) array.

2.1. Mathematical model

The kernel of a normal-square (90°) array is shown in Fig. 1. The diameter and pitch distances are represented by D and P respectively. The pitch ratio ($p^* = P/D$) is the same in both the longitudinal (in-flow) and transverse (flow-normal) directions. The inflow direction is shown by the bold arrows. The central cylinder is assumed to oscillate in the direction of the lift force only, designated here as flow normal direction. The schematic physical representation of the mass on a spring is shown in Fig. 1, where k, c stand for the cylinder stiffness and damping respectively. The mass per unit length of the cylinder is represented by m. The mass includes the hydrodynamic mass of the fluid medium at rest. Similarly the stiffness (k) and damping (c) coefficients are defined with respect to the quiescent fluid medium. Eq. (2) represents the motion of the cylinder in the flow normal direction. y is the instantaneous displacement of cylinder in this direction. t represents the time. The right hand term of the equation is a sinusoidal fluid force with an amplitude f_y per unit length of the cylinder and an angular periodicity (ω_{sh}) associated with the force.

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = f_y e^{-\hat{\imath}\,\omega_{sh}t} \tag{2}$$

where $\hat{\imath} = \sqrt{-1}$. Using the definitions of the natural angular frequency (ω_n) and damping ratio (ζ) of the cylinder, $\omega_n = \sqrt{k/m}$ and $\zeta = c/2\sqrt{km}$, Eq. (2) can be written as,

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n\frac{dy}{dt} + \omega_n^2 y = \frac{f_y}{m}e^{-\hat{\imath}\,\omega_{sh}t} \tag{3}$$

The general solution can be given as,

$$y = Ye^{-\hat{\imath}\,(\omega_{sh}t+\theta)} \tag{4}$$

where, Y is the magnitude of cylinder oscillations, while as θ is the phase difference between the fluid force and the cylinder response (y). The magnitude (Y) can be obtained by solving Eqs. (4) and (3).

$$-Y\omega_{sh}^2 e^{-\hat{\imath}\,\omega_{sh}t} + 2\zeta\omega_n Y(-\hat{\imath}\,\omega_{sh})e^{-\hat{\imath}\,(\omega_{sh}t+\theta)} + \omega_n^2 Ye^{-\hat{\imath}\,(\omega_{sh}t+\theta)} = \frac{f_y}{m}e^{-\hat{\imath}\,\omega_{sh}t}$$

$$-Y\omega_{sh}^2 + 2\zeta\omega_n Y(-\hat{\imath}\,\omega_{sh}) + \omega_n^2 Y = \frac{f_y}{m}e^{\hat{\imath}\,\theta}$$

By equating the real and imaginary parts, we can obtain,

$$(\omega_n^2 - \omega_{sh}^2)Y = \frac{f_y}{m}\cos(\theta) \tag{5}$$

$$(-2\zeta\omega_n\omega_{sh})Y = \frac{f_y}{m}\sin(\theta) \tag{6}$$

Thus,

$$Y = \frac{f_y/m}{\sqrt{(\omega_n^2 - \omega_{sh}^2)^2 + (2\zeta\omega_n\omega_{sh})^2}} \tag{7}$$

The unsteady response amplitude (Y) of the cylinder is directly proportional to the magnitude of the fluid force f_y and it is inversely proportional to the mass and damping terms. The phase difference (θ), between the fluid force acting on the cylinder and cylinder displacement is considered as an important component of the fluidelastic instability, particularly in the theoretical models based on Lever and Weaver (1982). The exact physics of the phase lag (θ) is not well understood (Khalifa et al., 2013). The phase lag is approximated by using an expression based on a hydraulic analogy in Lever and Weaver (1982). In Eq. (7), the phase lag (θ) is eliminated in the derivation of the displacement amplitude (Y), although its effect is incorporated in the square-root term.

The fluid force (f_y) in Eq. (7) can be expressed in terms of the pitch velocity (u_p) by an empirical relation as,

$$f_y = Eu_y\frac{1}{2}\rho u_p^2 D \tag{8}$$

where, Eu_y is an instantaneous component of the Euler number in the transverse direction. The Euler number in heat exchanger designs is commonly defined as,

$$Eu = \frac{\langle \Delta p_{row} \rangle}{\frac{1}{2}\rho \langle u_p \rangle^2} \tag{9}$$

where, Δp_{row} is an instantaneous pressure drop across a row of an array and $\langle \cdot \rangle$ represents ensemble averaging operation. An instantaneous Euler number in the flow direction can be given as,

$$Eu_x = \frac{\Delta p_{row}}{\frac{1}{2}\rho u_p^2} \tag{10}$$

Similarly, the flow normal component of the Euler number Eu_y is assumed to be based on the instantaneous pressure drop in the lift direction, Δp_y, across the cylinder. By using Eq. (8) in Eq. (7),

$$Y = \frac{Eu_y\frac{1}{2}\rho u_p^2 D}{m\omega_n^2\sqrt{\left(1-\left(\frac{\omega_{sh}}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega_{sh}}{\omega_n}\right)^2}} \tag{11}$$

The term in the square root acts as a mechanical impedance, which signifies the resistivity of the cylinder to the imposed harmonic force.