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Haar wavelet for solving the inverse point kinetics equations and estimation of feedback reactivity coefficient under background noise

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ABSTRACT

A new formalism is presented in this paper for solving the inverse point kinetics equations with six groups of delayed neutron precursors using Haar wavelet and estimation of feedback reactivity coefficient from the observed power transient under background noise. The Haar wavelet transforms the inverse point kinetics equations into a set of linear equations and these equations can be solved easily. Using this method the reactivity required for a desired power transient is obtained and also the feedback reactivity and the temperature coefficient of reactivity involved in the observed power transient are estimated for various background noise levels. This method is tested in two ways, i.e. (i) to estimate the reactivity required for different types of power transients in thermal reactor as well as in Indian Prototype Fast Breeder Reactor (PFBR) and (ii) to estimate the feedback reactivity and the temperature coefficient of reactivity involved in the power transients of thermal as well as Indian PFBR. In the case of Indian PFBR, the temperature coefficient of reactivity is estimated for various background noise levels in power transients. It is observed that as the noise level is reduced, the accuracy in the estimation of temperature coefficient of reactivity is increased. It is also shown that using Haar wavelet with beamforming, the temperature coefficient of reactivity can be estimated to a good accuracy even under high background noise. In this method the estimated feedback reactivity and the temperature coefficient of reactivity are found to be in good agreement with reference values. From the comparison of results it is observed that this method is efficient in estimating the reactivity required for different types of desired power transients in thermal as well as in fast reactors and this method is also efficient in estimating the temperature coefficient of reactivity under high background noise. This method is effective and simple to use.

1. Introduction

The time-dependent behavior of nuclear reactors is qualitatively described by the neutron density and the kind of reactivity acting on the reactor. Any deviation from the steady state behavior of nuclear reactor will result in the change of neutron density with time, which in turn will alter the power level. It is important from the safety point of view to monitor the neutron density and reactivity inside the reactor core during normal and accidental conditions. Generally the power transients, following any addition or removal of reactivity in small reactors, are investigated by solving the point kinetics equations with feedback. In the similar way the reactivity required for a desired power transient is obtained by solving the inverse point kinetics equations. For instance, in a postulated accidental scenario during reactor start-up, the uncontrolled movement of control rod may cause power transient and from the observed power transient, it is possible to estimate the kind of reactivity that might have caused the power transient, by solving the inverse point kinetics equations. The solution of inverse point kinetics equations with feedback is helpful in estimating the feedback reactivity coefficients acting on the reactor. The feedback reactivity coefficients are important in determining the average fuel pin temperature, clad and moderator temperatures. The peak fuel pin temperature decides the fuel integrity. The inverse point kinetics equations are also important in the design of reactivity meter, an instrument in nuclear power plant system, which calculates the reactivity acting on the reactor from the observed power transient. Generally background noise interferes in the measurement of power transients and in real-time scenario, it becomes equally important to solve the inverse point kinetics equations with feedback and background noise to estimate the kind of reactivity acting on the reactor.

The solution of inverse point kinetics equations requires the discretization of integral term associated with the precursor concentration. This discretization requires the power history (Shimazu et al., 1987; Hoogenboom and Van Der Slujis, 1988; Ansari, 1991). One can also use discrete Laplace transform technique to solve the inverse point kinetics equations (Diaz et al., 2008). Hamming method (Diaz et al., 2012) can

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also be used to solve the inverse point kinetics equations without requiring the power history. In this work, the inverse point kinetics equations are solved using Haar wavelet and the reactivity required for the desired power transient in thermal as well as in Indian Prototype Fast Breeder Reactor (PFBR) is estimated. By applying Haar wavelet, the inverse point kinetics equations are converted into linear equations and the linear equations can be solved with less computational effort to estimate the reactivity from the observed power transient. This method is used to estimate various types of reactivity perturbations, i.e. step, ramp, oscillatory and pulsed reactivity from the power transient. This method is also applied to estimate the temperature coefficient of reactivity from the power transients of thermal and Indian PFBR. In the case of Indian PFBR, the temperature coefficient of reactivity is estimated using synthetic power transients with various background noise levels. It is observed that as the background noise level is increased in the measurement of power transient, the accuracy in the estimation of temperature coefficient of reactivity is reduced and it is a natural phenomenon. But even in high background noise also, the temperature coefficient of reactivity can be accurately estimated by beamforming (Van Veen and Buckley, 1988) the power transients from different channels. Beamforming is done by averaging the signals from different channels with appropriate time delay. The beamforming technique reduces the noise. In the case of Indian PFBR, the synthetic power transients with high background noise levels are beamformed and the beamformed power transient is used to estimate the temperature coefficient of reactivity. In this case, while beamforming the power transients from different channels, it is assumed that there is no time delay. The estimated temperature coefficient of reactivity of the Indian PFBR, after beamforming, is found to be in good agreement with the reference value. It is also shown that as the number of channels in the beamforming is increased, the accuracy in the estimation of temperature coefficient of reactivity is improved. From the comparison of results, it is established that this method can be used to estimate the feedback reactivity and temperature coefficient of reactivity to a good accuracy from the observed power transients in thermal as well as in fast reactors with high background noise. This method is easy to implement and this method may be used in the real-time estimation reactivity from the observed power transient.

2. Inverse point kinetics equations

The point kinetics equations, describing the time evolution of reactor power for a desired reactivity insertion, $\rho(t)$, is given as (Duderstat and Hamilton, 1976; Bell and Glasstone, 1970).

$$\frac{dP(t)}{dt} = \left[\frac{\rho(t)-\beta}{\Lambda}\right]P(t) + \sum_{j=1}^{6}\lambda_j C_j(t) \tag{1}$$

$$\frac{dC_j(t)}{dt} = \frac{\beta_j}{\Lambda}P(t) - \lambda_j C_j(t), \quad j = 1,2,3...6 \tag{2}$$

In the above equation $P(t)$ is the power, $\rho(t)$ is the reactivity acting on the reactor, β_j is the effective fraction of j^{th} group of delayed neutrons, $C_j(t)$ is the j^{th} group of delayed neutron precursor density and β is the total fraction of delayed neutrons.

The reactivity $\rho(t)$ necessary for causing the desired power transient $P(t)$ can be obtained from the inverse point kinetics equation as (Duderstat and Hamilton, 1976).

$$\rho(t) = \beta + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{\Lambda}{P(t)}\sum_{j=1}^{6}\lambda_j C_j(t) \tag{3}$$

$$\frac{dC_j(t)}{dt} = \frac{\beta_j}{\Lambda}P(t) - \lambda_j C_j(t), \quad j = 1,2,3...6 \tag{4}$$

Solution of Eqs. (3) and (4) gives the reactivity required for the desired power transient $P(t)$. Here we make use of Haar wavelet for solving Eq. (4) and the solution is used in Eq. (3) to get the reactivity for the desired power transient $P(t)$.

3. Haar wavelet and solution of inverse point kinetics equations

In the recent years, the wavelet approach has become an important field in the numerical solution of differential equations. Different types of wavelets and approximating functions have been used in the numerical solution of initial and boundary value problems (Lepik 2005, 2007). Wavelets are basis functions, constructed from translation and dilation of mother wavelet. The scaling function for the family of the Haar wavelet, (in the interval [0, 1]) is defined as

$$h_1(t) = \begin{cases} 1 & t \in [0,1] \\ 0 & otherwise \end{cases} \tag{5}$$

The mother wavelet $h_2(t)$ is defined as ($t \in [0, 1]$)

$$h_2(t) = \begin{cases} 1 & t \in [0,\frac{1}{2}] \\ -1 & t \in [\frac{1}{2},1] \\ 0 & otherwise \end{cases} \tag{6}$$

All other wavelets can be generated using dilation and translation of the mother wavelet. In general, the Haar wavelet family can be defined for any time interval $t \in [\alpha, \beta]$. The integration of Haar functions is defined as

$$p_{i,1}(t) = \int_0^t h_i(t')dt' \tag{7}$$

The recurrence relation for $p_{i,\nu}(t)$ is given as

$$p_{i,\nu}(t) = \int_0^t p_{i,\nu-1}(t')dt', \quad \nu = 2,3.....$$

Any piecewise constant square integrable function $y(t)$, can be expanded using Haar wavelets (Lepik, 2005, 2007) as

$$y(t) = \sum_{i=1}^{2M} a_i h_i(t) \tag{8}$$

where a_i are the Haar wavelet coefficients, $M = 2^J$, J is the maximum order of resolution of wavelet and the collocation points are defined as

$$t_j = \frac{j-0.5}{2M}, \quad j = 1,2,3...2M \tag{9}$$

Haar wavelets are effective in solving ordinary and partial differential equations (Lepik, 2009). Here we apply the Haar wavelet in solving the inverse point kinetics equations for reactivity calculation. The precursor concentration equation, Eq. (4), is expanded into Haar wavelets as

$$\frac{dC_j(t)}{dt} = C_j' = \sum_{i=1}^{2M} b_i^j h_i(t) \tag{10}$$

The initial conditions are given as

$$P(t=0) = P(0)$$

$$C_j(0) = \frac{\beta_j}{\Lambda\lambda_j}P(0)$$

From Eq. (10) the delayed neutron precursor concentration $C_j(t)$ can be obtained as

$$C_j(t) = \sum_{i=1}^{2M} b_i^j p_{i,1}(t) + C_j(0) \tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (4) we get

$$\sum_{i=1}^{2M} b_i^j h_i(t) = \frac{\beta_j}{\Lambda}P(t) - \lambda_j\left(\sum_{i=1}^{2M} b_i^j p_{i,1}(t) + C_j(0)\right) \tag{12}$$

Using Haar wavelets, the equation governing the precursor concentrations (Eq. (4)) is converted into linear equations (Eq. (12)) and