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Modeling large number of rod-to-rod and rod-to-rigid surface frictional contact

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ABSTRACT

In many engineering systems such as nuclear fuel rods and heat exchangers tubes, mechanical components are subjected to a large number of frictional-contact constraints. An efficient and robust numerical scheme is needed for handling such a number of constraints. In this paper a numerical method is developed and presented to handle a large number of rod-to-rod and rod-to-rigid frictional contact constraints. New auxiliary incremental displacement variables are defined and the non-linear frictional-contact problem is formulated to be a linear complementarity problem (LCP). Use of LCP eliminate the need for iteration and satisfies all the frictional contact constraints in the whole system simultaneously. The mixed three-node beam finite elements are employed to model the longitudinal and lateral deformation of rods. The equations of motion of the entire dynamical system are discretized in the time-domain by means of the Bozzak-Newmark scheme. Validation cases are discussed and numerical results were obtained and presented for an array of 37 rods inside a tube.

1. Introduction

Frictional contact constraints are encountered in various fields of engineering such as a bundle of nuclear fuel rods inside a pressure tube, an array of hot tubes in a steam generator, a bunch of rollers in a sliding bearing, etc. Fretting and wear are particular types of surface damage caused by the mechanical interactions between components. Consideration and predicting of fretting and wear is of high importance in the design and maintenance of nuclear reactors. Simulating dynamic behavior of fuel rods and estimating fretting is a very challenging task and require an efficient numerical methodology. The focus of this study is to present a robust and efficient numerical method for handling such systems with large number of Unilateral Frictional Contact (UFC) constraints.

Park et al. (2011) studied the vibrational behavior of a fuel rod supported by elastoplastic supports. They have utilized a bilinear elastoplastic contact force model to simulate the nonlinear vibrational behaviour. Hassan and Rogers (2005) investigated vibration of a single fuel rod subjected to turbulence excitations. They applied several frictional models to understand the effect of tube-support clearance and preload on the predicted work rate. Work by Hassan and Rogers (2005), later on was improved by Mohany and Hassan (2013) to consider 3 locations of contact between the fuel rod and the pressure tube and to account for the effect of contact from neighbouring fuel rods. They applied turbulence and seismic excitation and found the values of work rate. Wear damage is quantified in terms of the removed volume of material. According to Archard's wear model (Suh, 1989), the rate of volume removal at a location of contact between two components is related to the sliding velocity and normal force. With the model presented in this paper, sliding displacement, velocities and the normal forces can be calculated at several contact locations simultaneously. Then the results may be used to estimate fretting and wear.

Due to compactness, the total number of potential frictional contact constraints can be very large. Solving such multibody contact problem is a very challenging task. Iterative-based methods and the penalty method are commonly used in solving frictional contact problems among beams. Xuewen et al. (2000) presented a non-smooth model for formulating frictional contact problem. Zavarise and Wriggers (2000) formulated a 3D finite element model to deal with contact and friction between straight beams by employing the penalty method. Neto et al. (2014) presented a new methodology to simulate the contact between a circular rod and a flat surface with the consideration of the rigid body rotation. They modified the classical tangential gap function in order to account for the rolling motion and the moment caused by the friction force.

More recently Litewka (2015) presented a 3D contact finite element formulation to model beam-to-beam contact with friction. He used the penalty method to enforce the contact and frictional constraints, and introduced two additional sets of contact points in situations where contact cannot be considered as point-wise or node-to-node contact. In

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the case of beam-to-rigid body contact problems several researchers used iterative methods in connection with the finite element analysis to solve the problem numerically. It is also reported that Mortar method with the penalty enforcement of contact constraints and the Mortar method with the Lagrange multiplier approach are also effective in handling the beam-to-beam and beam-to-rigid body contact problems Konyukhov and Schweizerhof (2015). According to Xuewen et al. (2000), in the penalty methods, ill-conditions occur frequently especially when the penalty factors are large. Finding the acceptable penalty parameters that give an accurate assessment of contact forces and at the same time makes the numerical model less prone to ill-posedness is a very challenging task. Mechanical systems are often subjected to large number of frictional-contact constraints and formulating a problem with large number of DOFs and contact constraints using the discussed methods is not efficient if not possible sometimes.

Another method for handling contact and friction that have received a wide attention is the linear complementarity problem (LCP) formulation. Yu and Hojatie (2013) presented an effective scheme based on LCP formulation for handling frictionless contact among an array of parallel rods for a static problem. Another challenge in dealing with a frictional contact problem is that the direction of contact and direction of the contact induced frictional force are not known a priori. In this paper, the direction of contact is defined in the direction of the closest distance between the two contacting points. For the direction of frictional force the procedure proposed by Fadaee and Yu (2015) is implemented.

In this study a numerical method based on Newmark and LCP formulation is presented to handle rod-to-rod and rod-to-rigid body contact. An implicit incremental displacement Bozzak-Newmark scheme is employed to seek a numerical solution in the time domain for the rods subjected to friction and unilateral contact constraints. The finite element model presented by Yu and Fadaee (2012) is employed to capture bending, axial, and torsional displacement of an individual rod using the classical theory of bending and longitudinal deformations.

In handling the multiple unilateral frictional constraints at a time step, the sub-structuring method is used to eliminate all interior DOF's (Yu and Hojatie, 2013). The coupled gap equations in the directions of all potential contact points and the corresponding frictional forces in the two tangential directions (axial and circumferential) are reduced through a coordinate transformation and an auxiliary incremental displacement variable, to a LCP for which a solution can be obtained using the Lemke algorithm. At each time step, the incremental displacement vectors are resolved into the tangential and normal directions of motion. Based on Coulomb's law of friction, the frictional force acts in the direction opposite to the true direction of motion or tendency of motion. The main advantage of the LCP is that at every time step, the solution satisfies all the contact and frictional constraints simultaneously without iterations. At the end, the presented numerical method is employed to obtain the free fall response of an array of 37 rods inside a tube with rigid boundaries. The mechanical system has 156 sets of potential rod-to-rod and 54 sets of rod-to-rigid contact constraints.

2. Dynamic equations of rods

An array of rods placed in a cylindrical tube is displayed in Fig. 1. Rods are modelled as deformable bodies and wall of the tube is considered rigid. Rods are closely packed and may potentially contact neighbouring rods or the wall of the tube through small pads that are attached to them at different axial locations (see Figs. 2 and 3). In a CANDU nuclear fuel bundle these pads are designed to 1) prevent direct contact between neighbouring rods and between rods and the tube, 2) promote heat transfer, 3) avoid creation of localized hot spot. From now on in this study, contacts between rods will be referred as internal contact and contact between rods and the tube will be referred as external contact. For simplicity, in this paper we have assumed that all internal contacts are frictionless while all external contacts have

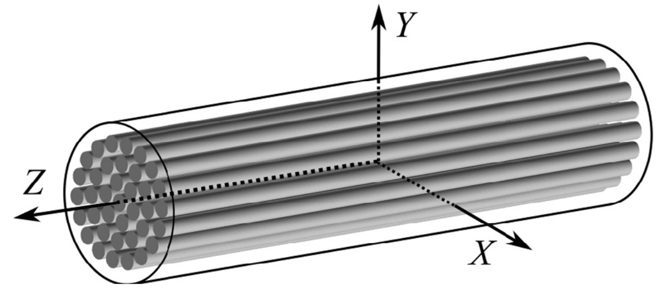

Fig. 1. An array of rods inside a tube.

friction. Contact/impact, stick and slip motion may occur at different contact locations when rods are interacting with each other and the tube.

Using finite element method equation of motion for an array of rods may be written as

$$[m]\{\ddot{q}\} + [c]\{\dot{q}\} + [k]\{q\} = \{Q\} + \{Q_f\} - \{Q_c\} \tag{1}$$

where $[m]$, $[k]$ and $[c]$ are the mass, stiffness and damping matrices respectively; $\{Q\}$ is the external force vector; $\{Q_c\}$ is the contact force vector; $\{Q_f\}$ is the friction force vector. Mass, stiffness and damping matrices are found by employing the finite element model presented by Yu and Fadaee (2012).

To find the numerical solution to the Eq. (1) the time domain is discretized into l equal steps with a time step of Δt as $t_i = t_0 + i\Delta t$, $i = 0, 1, 2, \ldots, l$. If the solution of the dynamical system is known for the time $t = t_i$ the state of the system at $t = t_{i+1}$ may be found by solving the following equation

$$\begin{aligned}(1+\alpha)[m]\{\ddot{q}\}_{i+1} - \alpha[m]\{\ddot{q}\}_{i+1} + [c]\{\dot{q}\}_{i+1} + [k]\{q\}_{i+1} \\ = \{Q\}_{i+1} + \{Q_f\}_{i+1} - \{Q_c\}_{i+1}\end{aligned} \tag{2}$$

where α is the relaxation factor. Using Newmark integration scheme, displacement, velocity and acceleration can be related as

$$\{\ddot{q}\}_{i+1} = \frac{1}{\beta\Delta t^2}(\{q\}_{i+1} - \{q\}_i - \Delta t\{\dot{q}\}_i - (0.5-\beta)\Delta t^2\{\ddot{q}\}_i) \tag{3}$$

$$\{\dot{q}\}_{i+1} = \{\dot{q}\}_i + (1-\gamma)\Delta t\{\ddot{q}\}_i + \gamma\Delta t\{\ddot{q}\}_{i+1} \tag{4}$$

where γ and β are the Newmark coefficients, which can be chosen in the following range for numerical stability (Rao, 2010)

$$\alpha \leqslant 0.5, \beta \geqslant \frac{\gamma}{2} \geqslant 0.25, \alpha + \gamma \geqslant 0.25 \tag{5}$$

In this study, the following values are used: $\alpha = 0.1$, $\beta = 0.5$ and $\gamma = 0.6$. Substitute Eqs. (3) and (4) into Eq. (2) one may arrive at

$$[k^*]\{\Delta q\}_{i+1} = \{Q^*\}_{i+1} + \{Q_f\}_{i+1} - \{Q_c\}_{i+1} \tag{6}$$

where $\{\Delta q\}_{i+1}$ is the incremental displacement and it may be defined as

$$\{\Delta q\}_{i+1} = \{q\}_{i+1} - \{q\}_i \tag{7}$$

and

$$[k^*] = (1+\alpha)\frac{1}{\beta h^2}[m] + \frac{\gamma}{\beta h}[c] + [k] \tag{8}$$

$$\begin{aligned}\{Q^*\}_{i+1} = \{Q\}_{i+1} + [m]\left(\frac{1+\alpha}{\beta h^2}\{q\}_i + \frac{1+\alpha}{\beta h}\{\dot{q}\}_i + \left(\frac{1}{2\beta} - 1 + \frac{\alpha}{2\beta}\right)\{\ddot{q}\}_i\right) \\ + [c]\left(\frac{\gamma}{\beta h}\{q\}_i + \left(\frac{\gamma}{\beta} - 1\right)\{\dot{q}\}_i + \left(\frac{\gamma}{2\beta} - 1\right)h\{\ddot{q}\}_i\right) - [k^*]\{q\}_i\end{aligned} \tag{9}$$

All the nodes in the mechanical system may be divided into two subsets: interior and interfacial nodes. Interior node's displacements are not involved explicitly in the contact formulations. Therefore the generalized force due to contact associated with interior DOF's are zero. On the other hand interfacial nodes are the nodes that potentially may be in contact, either internally with another node or externally with the