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A novel approach for bifurcation analysis of out of phase xenon oscillations using multipoint reactor kinetics

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ABSTRACT

In this paper, an approach for carrying out bifurcation analysis of out of phase xenon oscillations has been demonstrated using a reduced order model. The reduced order model consists of multipoint kinetic equations coupled with xenon and iodine equations along with a lumped power coefficient feedback. A large Pressurised Heavy Water Reactor (PHWR) has been considered. The reactor core is divided into two regions for the analysis to present a "proof of principle" for this approach. However, extension of this approach for multiple regions, for better accuracy, though not trivial is quite straight forward. Bifurcation analysis has been carried out using nonlinear analysis tools like MATCONT and BIFOR considering the operating power level of operation, power coefficient of reactivity and reactor size as bifurcation parameters. The results show the existence of Generalized Hopf (GH) points which divides the parameter space into subcritical and supercritical Hopf bifurcation regions. The identification of subcritical Hopf bifurcation is important as it shows unstable limit cycles in the region identified as stable by linear stability analysis.

1. Introduction

Instabilities due to oscillation in reactor power caused by the variation of xenon (Xe-135) have been of deep interest for reactor physicists from the beginning. The effects of xenon are a potential source of instability in thermal reactors due to its large thermal absorption microscopic cross-section. The instantaneous production rate of Xe-135 mainly depends upon the Iodine-135 concentration present at that moment and a small fraction is produced directly from fission. On the other hand, the removal of Xe-135 is mainly through the neutron absorption process which depends on the instantaneous flux and also through beta decay of Xe-135 with a half-life of 9.2 h. The delay between xenon removal and xenon buildup from iodine decay can establish an oscillatory regime in reactor power which can cause the regional powers to exceed the allowable limits.

These power oscillations due to variation of xenon can be in-phase or out of phase. During in-phase oscillations, the reactor power changes equally in phase in every point of the core leading to total power oscillations. However, in out of phase oscillations, local changes in xenon concentration due to local changes in power can lead to slow power oscillations. These oscillations may be induced by a change in the reactivity at some local region. The growth or decay of this type of out of phase oscillations depend on the size of the reactor, the operating power level, the extent of power flattening, magnitude of power coefficient and amount of spatial power control. Large thermal reactors operating at high flux levels are inherently unstable w.r.t. xenon induced spatial oscillations in the absence of any spatial control.

1.1. Previous work

The study of xenon oscillations are generally carried out using neutron diffusion theory codes and neutron kinetics. Initial work for space independent xenon oscillations was done by Chernick (1960) in which the phenomena of space independent (in-phase) xenon oscillations was analysed using solution of neutron point kinetic equations coupled with xenon and iodine equations for different parameter spaces. The analysis confirmed the existence of stable and unstable limit cycle solutions for certain parameter values. Also, the effect of temperature on xenon oscillations (Chernick et al., 1961) was studied and the linear stability boundaries in different parameter planes were estimated. This model was further extended by Lellouche (1962) to explicitly include spatial dependence for solid and annular cylindrical surfaces. Modal synthesis method for studying spatial oscillations for a slab reactor was used by Gyorey (1962) and Canosa and Brooks (1966). Lambda modes were used as expansion functions for flux expansion by Stacey (1969) and the linear stability boundary between the steady

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state flux level and the sub-criticality associated with each mode was estimated.

A two point coupled reactor equation for studying the same for a slab reactor was used by Kobayashi and Yoshikuni (1982). An analytical model for studying xenon oscillation due to control rod movement in a pressurized water reactor was developed by Song et al. (Jae et al., 1999). In 1995, Chernick's model was used by Rizwan-uddin (1995) for carrying out nonlinear stability analysis for space independent xenon oscillations. Expansion methods for estimating nonlinear stability domains were analyzed by Yang and Cho (1992) for in phase xenon oscillations. Neutron point kinetics equations were also used by Zarei et al. (2016) for studying the stability of space independent xenon oscillations for VVER 1000 reactors. Assessment of stability of spatial xenon oscillations for WWER 1000 was also carried out by Parhizkari et al. (2015) which involved solution of two region multipoint kinetics equations. Monte Carlo methods have been also used recently for simulation of xenon oscillations in Advance Candu Reactors (Chang, 2016).

Designing a control system for suppressing the xenon oscillations is very important for stable operation of thermal reactors. The in-phase xenon oscillations (total power oscillations) can be controlled by controlling the total power of the reactor. However, in case of out of phase oscillations, the total reactor power does not change but the regional powers oscillate and hence local power control is required for suppressing them. Multipoint kinetics equations coupled with xenon and iodine equations were used by Tiwari et al. (1996) and Tiwari (1999) for modeling a large PHWR for designing an optimal linear regulator for control of xenon oscillations.

The stability analysis in the area of study of xenon oscillations is usually carried out using numerical simulations at various points in parameter space (Chernick, 1960; Chernick et al., 1961; Tiwari et al., 1996; Gyorey, 1962; Parhizkari et al., 2015). This approach is quite involved since it involves identification of a relevant parameter space and multiple runs using different parametric values and initial conditions to arrive at a "rough estimate" of stable and unstable regions. The Monte Carlo methods as discussed by Chang (2016), despite being accurate and able to handle the fuel rod heterogeneities, are very time consuming since it would require multiple simulations. Therefore, multimodal and multipoint methods (which are based on diffusion approximation) are quite commonly used as it leads to quite accurate solutions in reasonable computational effort. Also, linear stability analysis, carried out by Stacey (1969) is valid for "small" perturbations and it was carried out without considering any intermodal coupling. Hence, the system behaviour for "large" perturbations cannot be predicted by the linear stability analysis i.e., the system might exhibit "unstable" periodic solutions on the stable side and "stable" periodic solutions on the unstable side of the stability boundary. Hence, a detailed bifurcation analysis is required to estimate the "global" stability characteristics of the system. The existence of "hard" or subcritical bifurcation has to be carefully examined in the parametric space. In the subcritical Hopf region, the equilibrium point is linearly stable but due to the existence of an unstable limit cycle about this point; growing oscillations might be observed for large perturbations. But for "soft" or supercritical Hopf bifurcations, the stable region of the stability boundary is globally stable but stable limit cycles exist on the unstable side. Both types of bifurcations may exist for a system in different regions of parameter spaces, and hence the point where the cross-over takes place needs to be identified which can be only done by non-linear analysis methods.

1.2. Focus of the present analysis

Although, bifurcation analysis has been carried out for some simplified models, however, it has been limited to study of in-phase/space independent xenon oscillations (Rizwan-uddin, 1995; Zarei et al., 2016). In this paper, an approach for carrying out bifurcation analysis of out of phase xenon oscillations has been demonstrated using reduced order model. Reduced order models have previously been used for carrying out nonlinear stability analysis for Density Wave Oscillations (DWO) in Boiling Water Reactors (BWRs). These involve neutron modal kinetics equations coupled with thermal hydraulic equations as used by Karve et al. (1997) and Dokhane (2004) and a review of various approaches used for the same is presented in Rizwan-Uddin (2006). The first two lambda modes (fundamental mode and the first azimuthal modes) were used for the expansion of the neutron flux. These models did not include xenon and iodine equations as the DWO phenomena takes place at a faster time scale compared to xenon oscillations. In the present analysis, the reduced order model consists of neutron multipoint kinetic equations coupled with xenon and iodine equations along with a lumped power coefficient feedback. Using this model, one can identify the stable and unstable regions of parameter space using linear stability analysis. Furthermore, this model can be utilized for performing nonlinear stability analysis. It is worth mentioning here that nonlinear stability analysis of out of phase xenon oscillations has not been ever carried out using multipoint kinetics.

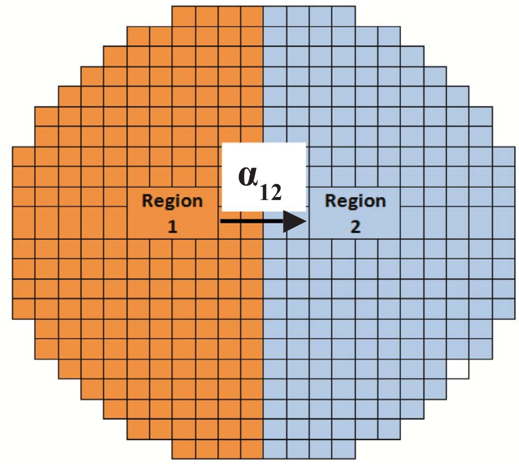

Fig. 1. A PHWR core divided into two regions.

A case for a large PHWR is considered for demonstrating this approach and the nonlinear stability characteristics of the out of phase xenon oscillations are estimated using codes BIFOR and MATCONT. BIFOR (Bifurcation Formulae for Ordinary-differential system) is a semi analytical bifurcation code developed by Hassard (1981) which calculates the bifurcation parameters like the critical value of the parameter, type of bifurcation, frequency of oscillation, etc. MATCONT (Govaerts, 2011) is a user friendly software based on method of numerical continuation which identifies the Generalized Hopf point, which separates subcritical Hopf region from the supercritical Hopf region.

The reactor core is divided into two regions for the analysis to present a "proof of principle" for this approach. However, extension of this model for more regions for better accuracy is fairly straight forward. The stability maps are constructed in different parameter spaces for out of phase xenon oscillations and numerical simulation of trajectories are done for verifying the stability boundaries. This type of non-linear stability analysis in different parameter spaces has not been carried out for out of phase xenon oscillations for PHWRs.

2. Mathematical modeling of a PHWR

2.1. Neutron multipoint kinetic equations

The main assumption in neutron point kinetics equations is that the reactor is treated to be a point. In order to study the spatial variation of variables, the reactor can be divided into smaller regions which interact through neutron diffusion which is reflected as coupling coefficients