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Novel arithmetic operations on type-2 intuitionistic fuzzy and its applications to transportation problem

Dipak Kumar Jana*

Department of Applied Science, Haldia Institute of Technology, Haldia, Purba Midnapur, 721657, West Bengal, India

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A B S T R A C T

Type-2 intuitionistic fuzzy sets possess many advantages over type-1 fuzzy sets because their membership functions are themselves fuzzy, making it possible to model and minimize the effects of uncertainty in type-1 intuitionistic fuzzy logic systems. This paper presents generalized type-2 intuitionistic fuzzy numbers and its different arithmetic operations with several graphical representations. Basic generalized trapezoidal intuitionistic fuzzy numbers considered for these arithmetic operations are formulated on the basis of (α, β)-cut methods. The ranking function of the generalized trapezoidal intuitionistic fuzzy number has been successively calculated. To validate the proposed arithmetic operations, we solved a type-2 intuitionistic fuzzy transportation problem by the ranking function for mean interval method. Transportation costs, supplies and demands of the homogeneous product are type-2 intuitionistic fuzzy in nature. A numerical example is presented to illustrate the proposed model.

1. Introduction

To make a transportation plan for the next day, the supply capacity at each origin, the demand at each destination and the conveyance capacity often need to be estimated by the professional judgement of experts or probability statistics because no precise a priori information exists. Certain hidden costs, such as toll tax and service tax, must be considered during transport. It is appropriate to investigate this problem by using fuzzy or stochastic optimization methodologies. The applicable theoretical methods can be referred to as fuzzy set theory and type-2 intuitionistic fuzzy sets. Real-life decision making problems display some level of imprecision and vagueness in estimation of parameters. Results have been captured by fuzzy sets modelling the problems. Applications of fuzzy set theory in decision making and in particular optimization problems have been widely studied since the introduction of fuzzy sets cf. Zadeh [13]. Recently, many papers have shown growing interest in the study of decision making problems using intuitionistic fuzzy sets/numbers [1,2]. The intuitionistic fuzzy set (IFS) is an extension of fuzzy set. IFS was first introduced by Atanassov [4]. The conception of IFS can be viewed as an approach where given data are not sufficient to define the fuzzy set. Fuzzy sets are characterized by the membership function only, but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [5]. Presently IFSs are being studied and used in different fields of science and technology for decision making problems. Several researchers have formulated and solved optimization problems in the field of intuitionistic fuzzy [6,8,9,14,20].

In the study of fuzzy set theory for optimization, the ranking of fuzzy numbers is a significant factor. To rank fuzzy numbers, one fuzzy number needs to be compared with the others by using a ranking function. Some researchers have formulated and solved optimization problems in the application of a ranking function [3,7]. Recently, the IFN received wide attention. Different definitions of IFNs have been proposed with corresponding ranking functions. Some research has also shown interest in the arithmetic operations and the ranking functions of IFNs [1,2].

Recently, IFNs have been used in fields, such as fuzzy linear programming and transportation problems. Parvathi et al. [10] have proposed an intuitionistic fuzzy simplex method. Pramanik et al. [16], Chakraborty et al. [17], Jana et al. [18], Jana et al. [19], Hussain et al. [11] and Nagoor Gani [12] proposed a method for solving intuitionistic fuzzy transportation problems. None of them introduced the generalized intuitionistic fuzzy number and its application to transportation problems.

* Fax: +91 3224 252800.
E-mail address: dipakjana@gmail.com.
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The membership functions of type-2 fuzzy sets are themselves fuzzy. Type-2 fuzzy sets are nowhere near as widely used as type-1 fuzzy sets. In 1975, the basic concept of type-2 fuzzy sets (T2FSs) was proposed by professor Zadeh [24] that is an extension of ordinary fuzzy sets i.e., type-1 fuzzy sets, whose truth values are ordinary fuzzy sets, i.e., fuzzy truth values. The overviews of type-2 fuzzy sets were given in Mendel et al. [21]. Since ordinary fuzzy sets and interval-valued fuzzy sets are special cases of type-2 fuzzy sets, Takac [22] proposed that type-2 fuzzy sets are very practical in circumstances where there are more uncertainties. Pramanik et al. [16] proposed type-2 fuzzy Gaussian fuzzy sets from the view of type reduction and centroid [23].

This paper presents generalized intuitionistic fuzzy arithmetic operations and their application to a transportation problem. Trapezoidal type-2 intuitionistic fuzzy numbers (TrT2IFN) are defined, and their arithmetic operations based on the type-2 intuitionistic fuzzy extension principle and $(\alpha,\beta)-cut$ method are presented. To illustrate the proposed method, a numerical example is presented and solved as a type-2 intuitionistic transportation problem.

2. Preliminaries

In this section, we first discuss the arithmetic operations on intuitionistic type-2 fuzzy sets with graphical representation. Next, we provide a number of definitions and notations for convenience of explaining general concepts concerned with intuitionistic type-2 fuzzy sets.

Definition 2.1. ***Generalized Intuitionistic Fuzzy Number (GIFN):*** *An Intuitionistic fuzzy number $\tilde{A}^I=\{<x,\mu_{\tilde{A}},\nu_{\tilde{A}}>\}$ of the real line $\Re$ is called GIFN, if the following hold*

(i) *there exists $x\in\Re$, $\mu_{\tilde{A}}(m)=w$, $\nu_{\tilde{A}}(m)=0$, $0<w\le 1$.*
(ii) *$\mu_{\tilde{A}}$ is continuous mapping from $\Re$ to the closed interval $[0,w]$ and $x\in\Re$, the relation $0\le\mu_{\tilde{A}}(x)+\nu_{\tilde{A}}(x)\le w$ holds.*

The membership function and non-membership function of $\tilde{A}^I$ is of the following form

$$\mu_{\tilde{A}}(x)=\begin{cases} wf_1(x), & m-\alpha\le x\le m;\\ w, & x=m;\\ wh_1(x), & m\le x\le m+\beta;\\ 0, & otherwise.\end{cases}$$

The non-membership function is of the following form

$$\nu_{\tilde{A}}(x)=\begin{cases} wf_2(x), & m-\alpha'\le x\le m, 0\le w(f_1(x)+f_2(x))\le w;\\ 0, & x=m;\\ wh_2(x), & m\le x\le m+\beta', 0\le w(h_1(x)+h_2(x))\le w;\\ w, & otherwise.\end{cases}$$

In this equation, $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing functions in $[m-\alpha,m]$ and $[m,m+\beta]$, respectively, and $f_2(x)$ and $h_2(x)$ are strictly decreasing and increasing functions in $[m-\alpha',m]$ and $[m,m+\beta']$ respectively, where m is the mean value of $\tilde{A}^I$. The left and right spreads of membership function $\mu_{\tilde{A}}(x)$ are called α and β. The left and right spreads of non-membership function $\nu_{\tilde{A}}(x)$ are called α' and β'.

Definition 2.2. ***Generalized Trapezoidal Intuitionistic Type-2 Fuzzy Number (GTIT2FN):*** *Let $\zeta\in\{L,U\}$ and $a_1'^{\zeta}\le a_1^{\zeta}\le a_2^{\zeta}\le a_3^{\zeta}\le a_4^{\zeta}\le a_4'^{\zeta}$. A GTIT2FN $\tilde{A}^I=[\mu^L_{\tilde{A}}(x),\mu^U_{\tilde{A}}(x),\nu^L_{\tilde{A}}(x),\nu^U_{\tilde{A}}(x)]$ in $\Re$ written as $(a_1^{\zeta},a_2^{\zeta},a_3^{\zeta},a_4^{\zeta};w^{\zeta})(a_1'^{\zeta},a_2^{\zeta},a_3^{\zeta},a_4'^{\zeta};w^{\zeta})$ has membership function (in Fig. 1).*

$$\mu^{\zeta}_{\tilde{A}}(x)=\begin{cases} w^{\zeta}\dfrac{x-a_1^{\zeta}}{a_2^{\zeta}-a_1^{\zeta}}, & a_1^{\zeta}\le x\le a_2^{\zeta};\\ w^{\zeta}, & a_2^{\zeta}\le x\le a_3^{\zeta};\\ w^{\zeta}\dfrac{a_4^{\zeta}-x}{a_4^{\zeta}-a_3^{\zeta}}, & a_3^{\zeta}\le x\le a_4^{\zeta};\\ 0, & otherwise.\end{cases}$$

and non-membership function

$$\nu^{\zeta}_{\tilde{A}}(x)=\begin{cases} w^{\zeta}\dfrac{a_2-x}{a_2^{\zeta}-a_1'^{\zeta}}, & a_1'^{\zeta}\le x\le a_2^{\zeta};\\ 0, & a_2^{\zeta}\le x\le a_3^{\zeta};\\ w^{\zeta}\dfrac{x-a_3^{\zeta}}{a_4'^{\zeta}-a_3^{\zeta}}, & a_3^{\zeta}\le x\le a_4'^{\zeta};\\ w^{\zeta}, & otherwise.\end{cases}$$

Definition 2.3. ***Generalized Triangular Intuitionistic Type-2 Fuzzy Number (GTrIT2FN):*** *Let $\zeta\in\{L,U\}$ and $a_1'^{\zeta}\le a_1^{\zeta}\le a_2^{\zeta}\le a_3'^{\zeta}$. A GTIT2FN $\tilde{A}^I=[\mu^L_{\tilde{A}}(x),\mu^U_{\tilde{A}}(x),\nu^L_{\tilde{A}}(x),\nu^U_{\tilde{A}}(x)]$ in $\Re$ written as $(a_1^{\zeta},a_2^{\zeta},a_3^{\zeta};w^{\zeta})(a_1'^{\zeta},a_2^{\zeta},a_3'^{\zeta};w)$ has membership function:*

$$\mu^{\zeta}_{\tilde{A}}(x)=\begin{cases} w^{\zeta}\dfrac{x-a_1}{a_2^{\zeta}-a_1^{\zeta}}, & a_1^{\zeta}\le x\le a_2^{\zeta};\\ w^{\zeta}, & x=a_2^{\zeta};\\ w^{\zeta}\dfrac{a_3^{\zeta}-x}{a_3^{\zeta}-a_2^{\zeta}}, & a_2^{\zeta}\le x\le a_3^{\zeta};\\ 0, & otherwise.\end{cases}$$

and non-membership function

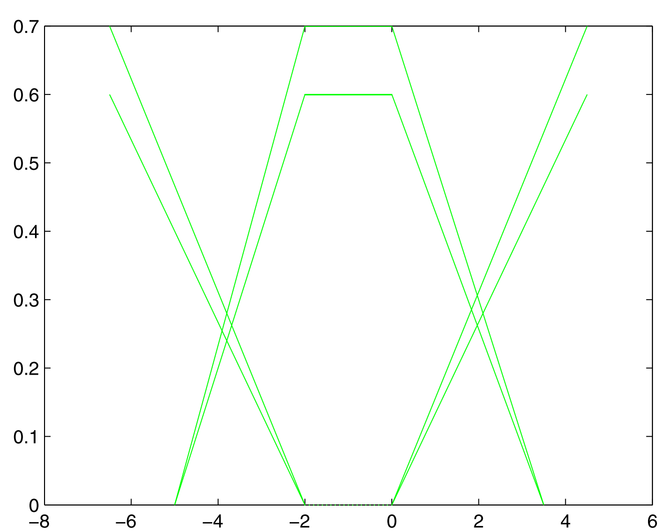

Fig. 1. Membership and non-membership function of GTIT2FN.