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Aspiration level approach to solve matrix games with I-fuzzy goals and I-fuzzy pay-offs

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ABSTRACT

The objective of this paper is to develop a new solution methodology for matrix games, in which goals are viewed as intuitionistic fuzzy sets (IFSs) and the elements of the pay-off matrix are represented by triangular intuitionistic fuzzy numbers (TIFNs). In this methodology, a suitable ranking function is defined to establish an order relation between two TIFNs, and the concept of intuitionistic fuzzy (I-fuzzy) inequalities is interpreted. Utilizing these inequality relations and ranking functions, a pair of linear programming models is derived from a pair of auxiliary intuitionistic fuzzy programming models. Based on the aspiration levels, this pair of linear programming models is solved to determine the optimal strategies for both players of the game. The proposed method in this paper is illustrated with a voting share problem to demonstrate the validity and applicability of the method.

1. Introduction

In traditional game theory, the precise assessment information is difficult due to a lack of information about the exact values of certain parameters and the fuzzy understanding of various situations by players. Fuzzy set theory, which is a very useful tool in game theory, has achieved a substantial amount of success. (Campos [9], Nishizaki and Sakawa [27,28], Bector and Chandra [8], Nayak and Pal [21,22], Li [14,16], Vidyottama and Chandra [37], Vijay et al. [38,39], Liu and Kao [18], Cevikel and Ahlatcioglu [10], Kacher and Larbani [12], Seikh et al. [32,33]). However, a fuzzy set only employs a membership degree. The degree of non-membership is automatically equal to the complement to 1. In real situations, however, players/decision makers often do not express the degree of non-membership as the complement to 1. Players/decision makers may exhibit some degree of hesitation. Therefore, a fuzzy set has no means of incorporating a degree of hesitation.

The intuitionistic fuzzy set (IFS), which is a generalization of fuzzy set theory, was introduced by Atanassov [4,5]; it is suitable for solving problems concerning vagueness. An IFS is characterized by two functions—a function that expresses the degree of membership and a function that expresses the degree of non-membership—thus, the sum of both values is less than or equal to 1. The degree of hesitation is equal to one minus the sum of the degree of membership and the degree of non-membership. Therefore, the concept of the IFS is considered to be an alternative approach for defining a fuzzy set in cases where available information is insufficient for defining an imprecise concept by conventional fuzzy sets. Therefore, an IFS can be employed to simulate the human decision-making process and any activity that requires human expertise and knowledge, which are inevitably imprecise or not completely reliable. Atanassov [6] discussed an open problem about the interpretation of an IFS in different optimization problems. Angelov [3] was the first researcher to answer this problem by implementing an optimization technique in an intuitionistic fuzzy environment. However, an IFS can be applied to game problems as the players have some degree of hesitation about appropriate pay-off values and the selection of a strategy for each of the pay-offs.

Intuitionistic fuzziness in matrix games is generally applied using two methods: in the first method, players have I-fuzzy goals; in the second method, the elements of the pay-off matrix

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are represented by intuitionistic fuzzy numbers (Li [13], Seikh et al. [30]). Recent studies have focused on the application of an IFS to resolve game problems. Atanassov [6] described a game problem using an IFS. Li and Nan [17] developed a nonlinear programming approach to matrix games with pay-offs of Atanassov's IFS. Aggarwal et al. [1,2] extended the duality results for two-person zero-sum matrix games with fuzzy goals and fuzzy pay-offs to an I-fuzzy scenario. Nan et al. [20] implemented a lexicographic method for matrix games in which pay-offs are represented by TIFNs. Li et al. [15] employed a bi-objective programming approach to solve a matrix game with pay-offs of TIFNs. Bandyopadhyay et al. [7] solved matrix games with intuitionistic fuzzy pay-offs using a score function. Nan and Li [19] outlined a linear programming approach to solve matrix games with I-fuzzy goals. Seikh et al. [31,34,36] investigated matrix games in which goals are represented as an IFS or elements of a pay-off matrix are represented by TIFNs. Nayak and Pal [23,24] constructed auxiliary linear programming models to solve bi-matrix games with goals expressed by an IFS. Seikh et al. [29,35] applied TIFNs to bi-matrix games. No study has investigated matrix games with I-fuzzy goals and I-fuzzy pay-offs. In this paper, a new methodology, in which goals are represented as an IFS and elements of a pay-off matrix are represented by TIFNs, is introduced to solve matrix games. The idea of double I-fuzzy inequalities, i.e., the I-fuzzy inequalities and the parameters that are represented by I-fuzzy numbers, is outlined. The concept of TIFNs and their arithmetic operations and cut sets are recalled. A new order relation is proposed to rank the two TIFNs. A pair of linear programming models is derived from a pair of auxiliary I-fuzzy programming models using these ranking order relations. These two models are solved by aspiration levels, and the optimal strategies for both players are obtained.

The paper is organized as follows: In Section 2, some definitions and preliminaries about an IFS and TIFNs are recalled and a ranking function is defined to establish an order relation between two TIFNs. Section 3 describes the application of an IFS in optimization and the concept of double I-fuzzy constraint conditions. The main problem about the matrix games with I-fuzzy goals and I-fuzzy pay-offs is formulated in Section 4. The solution procedure of these games is conceptualized by the degree of acceptance and the degree of rejection of the I-fuzzy aspiration levels for two players. The results are illustrated by considering a voting share problem in Section 5. Section 6 reflects the conclusions of this paper.

2. Definitions and preliminaries

2.1. Intuitionistic fuzzy sets

The intuitionistic fuzzy set, which was introduced by Atanassov [5], is characterized by two functions that express the degree of belongingness and the degree of non-belongingness.

Definition 1. Let $U = \{x_1, x_2, \cdots, x_n\}$ be a finite universal set. The Atanassov's intuitionistic fuzzy set (IFS) $\widehat{A}$ in a given universal set U is an object with the form

$$\widehat{A} = \left\{ \left\langle x,\ \mu_{\widehat{A}}(x),\ \gamma_{\widehat{A}}(x) \right\rangle \middle| x \in U \right\} \tag{1}$$

where the functions $\mu_{\widehat{A}} : U \to [0,1]$ and $\gamma_{\widehat{A}} : U \to [0,1]$ define the degree of membership and the degree of non-membership, respectively, of an element $x \in U$ to the set $A \subseteq U$ such that they satisfy the following conditions:

$$0 \le \mu_{\widehat{A}}(x) + \nu_{\widehat{A}}(x) \le 1, \quad \forall\ x \in U$$

which is known as an intuitionistic condition. The degree of acceptance $\mu_{\widehat{A}}(x)$ and the degree of non-acceptance $\nu_{\widehat{A}}(x)$ can be arbitrary.

Definition 2. Let $\widehat{A}$ and $\widehat{B}$ be two IFSs in the set U. Then, the intersection of $\widehat{A}$ and $\widehat{B}$ are defined as follows:

2.2. Triangular intuitionistic fuzzy number (TIFN)

In this section, the definitions are derived from Li [13].

Definition 3. (TIFN) The TIFN $\tilde{a} = \langle (a_l, a, a_r); w_a, u_a \rangle$ is a convex IFS on the set $\Re$ of real numbers with $a_l < a < a_r$, whose membership function and non-membership function are defined as

$$\mu_{\tilde{a}}(x) = \begin{cases} w_a \dfrac{x - a_l}{a - a_l} : if\ \ a_l \le x < a \\ w_a : if\ \ x = a \\ w_a \dfrac{a_r - x}{a_r - a} : if\ \ a < x \le a_r \\ 0 : if\ \ x < a_l\ or\ x > a_r \end{cases} \tag{2}$$

and

$$\nu_{\tilde{a}}(x) = \begin{cases} \dfrac{a - x + (x - a_l)u_a}{a - a_l} : if\ \ a_l \le x < a \\ u_a : if\ \ x = a \\ \dfrac{x - a + (a_r - x)u_a}{a_r - a} : if\ \ a < x \le a_r \\ 1 : if\ \ x < a_l\ or\ x > a_r \end{cases} \tag{3}$$

as depicted in Fig. 1.

The values w_a and u_a represent the maximum degree of membership and the minimum degree of non-membership, respectively, such that they satisfy the following conditions: $0 \le w_a \le 1, 0 \le u_a \le 1$ and $0 \le w_a + u_a \le 1$. Let $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - \nu_{\tilde{a}}(x)$, which is referred to as the I-fuzzy index of an element x in the TIFNa. It is the degree of indeterminacy membership of the element x to the TIFN $\tilde{a}$.

If $\mu_{\tilde{a}}(x) + \nu_{\tilde{a}}(x) = 1\ \forall\ x \in U$, then $\tilde{a} = \langle (a_l, a, a_r); w_a, u_a \rangle$ is reduced to $\tilde{a} = \langle (a_l, a, a_r); w_a, 1 - w_a \rangle$, which is a triangular fuzzy number (TFN). The definition of a TIFN is a generalization of the definition of the TFN, which was introduced by Dubois and Prade [11]. Two new parameters—w_a and u_a—are introduced to reflect the confidence level of a TIFN and the non-confidence level of a TIFN, respectively. Therefore, a TIFN may express more uncertainty compared with a TFN. The set of all TIFNs is denoted by $\tilde{F}(\Re)$.

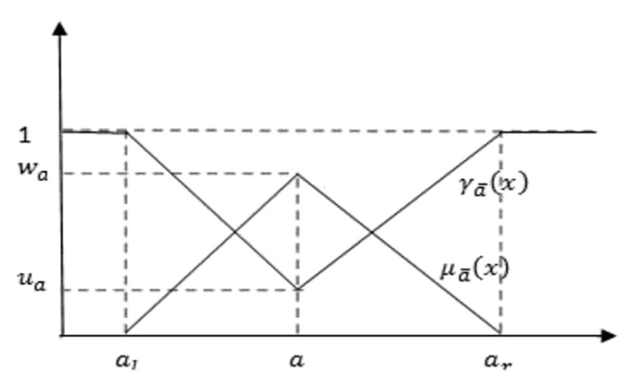

Fig. 1. Triangular intuitionistic fuzzy number.

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