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Transient heat conduction in an infinite medium subjected to multiple cylindrical heat sources: An application to shallow geothermal systems

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ABSTRACT

In this paper, we introduce analytical solutions for transient heat conduction in an infinite solid mass subjected to a varying single or multiple cylindrical heat sources. The solutions are formulated for two types of boundary conditions: a time-dependent Neumann boundary condition, and a time-dependent Dirichlet boundary condition. We solve the initial and boundary value problem for a single heat source using the modified Bessel function, for the spatial domain, and the fast Fourier transform, for the temporal domain. For multiple heat sources, we apply directly the superposition principle for the Neumann boundary condition, but for the Dirichlet boundary condition, we conduct an analytical coupling, which allows for the exact thermal interaction between all involved heat sources. The heat sources can exhibit different time-dependent signals, and can have any distribution in space. The solutions are verified against the analytical solution given by Carslaw and Jaeger for a constant Neumann boundary condition, and the finite element solution for both types of boundary conditions. Compared to these two solutions, the proposed solutions are exact at all radial distances, highly elegant, robust and easy to implement.

1. Introduction

Most of the currently utilized analytical solutions for heat equations in solid domains are based on the work provided by Carslaw and Jaeger [1], who seem to be the first to introduce a comprehensive treatment of heat conduction in solids subjected to different combinations of initial and boundary conditions. They introduced solutions to heat flow in finite, semi-infinite and infinite domains subjected to point, line, plane, spherical and cylindrical heat sources. In this paper, the focus is placed on heat flow in an infinite domain subjected to cylindrical heat sources, a topic which is central in many engineering applications, mainly in modeling shallow geothermal systems [2–5].

A shallow geothermal system, known as geothermal heat pump (GHP), and also ground source heat pump (GSHP), is a source of renewable energy that utilizes the earth heat energy from shallow depths for heating and cooling of buildings. It works by circulating a fluid in a borehole heat exchanger (BHE) which ensures a good thermal interaction with the surrounding soil mass. In many of the currently available models for shallow geothermal systems, the BHE is considered as a constant heat source.

Usually, shallow geothermal systems consist of multiple borehole heat exchangers. Modeling such a system typically requires numerical methods, such as the finite difference [6] and [7], finite volume [8] and [9], or finite element [10–12]. Nevertheless, some limited number of analytical and semi-analytical models has been introduced, notably those given by Eskilson and Claesson [13], Pasquier and Marcotte [14] and Erol et al. [15]. The basic idea behind the possibility of utilizing analytical methods for solving multiple heat sources problems is the use of the superposition principle.

Eskilson and Claesson [13] introduced a semi-analytical model for heat flow in a 1D finite line heat source embedded in an axisymmetric solid mass. They utilized the principle of superposition to account for multiple heat sources. They introduced what they termed “error” to approximate the difference between heat flow due to a single heat source and that of multiple heat sources. The approximation is made using the Fourier expansion to

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Nomenclature

T	Temperature of the solid mass (K)
$\widehat{T}$	Temperature frequency response
T_s	Heat source temperature(K)
T_{st}	Initial steady state temperature (K)
$q_s(t)$	Heat source flux (W/m)
q_o	Constant cylindrical heat flux per meter length
r	Radial distance(m)
r_s	Radius of the heat source (m)
α	Thermal diffusivity (m^2/s)
λ	Thermal conductivity (W/m·K)
ρ	Mass density (kg/m^3)
c	Specific heat capacity (J/kg·K)
I_o	First kind of modified Bessel functions
K_o	Second kind of modified Bessel functions
J_0	First kind Bessel functions of the order 0
J_1	First kind Bessel functions of the order 1
Y_0	Second kind Bessel functions of the order 0
Y_1	Second kind Bessel functions of the order 1
N	Number of the discrete samples
ω	Angular frequency

the first order of the thermal interaction between the heat sources. Their solution is effective for symmetric heat sources distribution, and necessitates separation between heat sources at the corners of the geometry and those in the middle.

Pasquier and Marcotte [14] introduced a semi-analytical model for heat flow in a solid mass subjected to multiple heat sources with time-varying heat fluxes and temperatures. The model allows for the imposition of heat sources with different heat fluxes or temperatures. They applied the fast Fourier transform for the temporal domain and the superposition principle for the spatial domain. The multiple heat sources system is solved using an iterative algorithm, which couples the thermal interaction between the involved heat sources. The algorithm has been applied to the infinite line source model, but can be extended to any model that can be decomposed into an incremental heat flux function, and the involved integral can be evaluated for a unit rectangular heat pulse, such as the finite line source and the infinite cylindrical line source.

Erol et al. [15] introduced a modified Green's function for heat flow in a porous domain subjected to a constant line heat source with a finite length. The prescribed heat flux is discontinuous, described by a rectangular pulses function. The convolution theory in time domain is utilized to solve the initial and boundary value problem for a single heat source. For the multiple heat sources, they utilized the superposition principle by summing up the temporal convolved functions of the heat sources.

In this paper, we elaborate on these models and introduce analytical solutions for transient heat flow in an infinite solid mass subjected to a varying single or multiple cylindrical heat sources. Solutions for two types of boundary conditions are introduced: a prescribed heat flux (Neumann boundary condition), and a prescribed temperature (Dirichlet boundary condition). We solve the initial and boundary value problem using the modified Bessel series and the fast Fourier transform. For multiple heat sources, we apply directly the superposition principle for the Neumann boundary condition. For the Dirichlet boundary condition, an analytical coupling. allowing for the thermal interaction between all involved heat sources, is conducted. The heat sources can exhibit different time-dependent signals, and can have any distribution in space.

2. Single heat source in a solid mass

Heat conduction in an infinite cylinder constituting a homogeneous, isotropic solid is described as

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}-\frac{\partial^2 T}{\partial r^2}-\frac{1}{r}\frac{\partial T}{\partial r}=0 \tag{1}$$

where $T=T(r)$ is the temperature of the solid mass; r is the radial distance: and α(m^2/s) is its thermal diffusivity, defined as

$$\alpha=\frac{\lambda}{\rho c} \tag{2}$$

in which λ(W/m K) is the thermal conductivity; ρ(kg/m^3) is the mass density; and c(J/kg K) is the specific heat capacity.

The initial condition is:

$$T(r,\ \ t=0)\ =\ T_{st} \tag{3}$$

in which T_{st} is the initial steady state temperature before operating the heat sources.

The boundary condition at infinity is:

$$\Delta T|_{r=\infty,t}=T|_{r=\infty,t}-T_{st}=0 \tag{4}$$

which implies that the heat source effect vanishes at far distances.

The boundary condition at the sources might be any of two types:

Neumann boundary condition:

$$-\lambda\frac{dT}{dr}\bigg|_{r=r_s}=q_s(t) \tag{5}$$

Dirichlet boundary condition:

$$T|_{r=r_s}=T_s(t) \tag{6}$$

where $q_s(t)$ is the heat source flux (W/m); $T_s(t)$ is the heat source temperature; and r_s is the radius of the heat source (for a line source, r_s approaches zero).

Applying Fourier transform of Eq. (1), gives [12].

$$\frac{i\omega}{\alpha}\widehat{T}-\frac{\partial^2\widehat{T}}{\partial r^2}-\frac{1}{r}\frac{\partial\widehat{T}}{\partial r}=0 \tag{7}$$

where $\widehat{T}$ is the temperature frequency response. Eq. (7) is a complex ordinary differential equation, describing a modified Bessel equation. The solution of this equation can be expressed as

$$\widehat{T}(r,\omega)=AK_o(k\,r)+BI_o(k\,r) \tag{8}$$

where

$$k=\sqrt{\frac{i\omega}{\alpha}} \tag{9}$$

and I_o and K_o are the first and second kind of modified Bessel functions.

Applying the boundary condition, Eq. (4), to Eq. (8), leads to

$$\Delta\widehat{T}\bigg|_{r=\infty}=AK_o(\infty)+BI_o(\infty)=0 \tag{10}$$

As $K_o(\infty)=0$ and $I_o(\infty)=\infty$, it implies that $B=0$, yielding

$$\widehat{T}(r,\omega)=AK_o(k\,r) \tag{11}$$