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The potential of cross-stream aligned sub-arrays to increase tidal turbine efficiency

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A theoretical model is proposed for a row of sub-arrays of tidal turbines aligned in a cross-stream fashion across part of a wide channel. This model builds on previous work investigating the behaviour of a single partial row array that split the problem into two flow scales; device and channel. In the present work, three flow scales are proposed: device, sub-array and channel flow, allowing the mass, energy and momentum conservation balance to be assessed separately at each scale. The power potential of a row of sub-arrays with varying blockage ratios at each flow scale is investigated, and it is found that increasing device local blockage has the greatest potential to increase power yield. It is also found that, for such a single row tidal farm with a sufficient number of devices in a very wide channel, splitting the long fence array into multiple smaller co-linear sub-fences can increase the overall energy extraction potential. A new maximum power coefficient is found in infinitely wide flow, increasing from the Lanchester-Betz limit of 0.593 for turbines in unblocked flow, past the partial row array limit of 0.798, to a new limiting value of 0.865 for a row of multiple sub-arrays.

1. Introduction

There is much interest at present in the potential of tidal turbines to address the need for a renewable energy source which is reliable and predictable. In the UK, tidal stream energy is identified as a key constituent of marine energy development to meet the 2020 energy targets [4] and there is considerable work being undertaken in the sector at present [16]. In order to generate a significant amount of power, it is widely accepted that large numbers of turbines will need to be grouped together in arrays or farms, most likely at sites with tidal flow regimes which are best suited to energy extraction. The initial industry assumption was that such tidal turbine farms would be arranged with turbines working independently of each other as in a wind farm, however this neglects the differing flow regimes between the open atmosphere and a constrained tidal channel. Recent work has indicated that significant improvements in power extraction can be gained by capitalising on the effect of channel blockage, which causes the flow through the turbines and the bypassing flow to interact to enable greater pressure (static head) drop across the turbine, resulting in increased power extraction. This effect can be achieved by arranging turbines in long row arrays, and it has been found [24,25] that in such cases the limiting maximum power which can be extracted from the free-stream energy flux can be significantly increased from that of a single turbine operating alone in unconstrained flow, known as the Lanchester-Betz limit, which is the applicable limit for a wind turbine.

The work of Lanchester and Betz in the early 20th century [1,15] derived an upper theoretical limit for power extraction from a turbine, equating to 16/27 or 0.593 of the upstream kinetic flux. This work assumed an infinite flow field, which is a reasonable assumption for a wind turbine in the unconstrained atmosphere but is less well-suited to tidal turbines in the relatively constrained environment of coastal waters. When interest in tidal stream turbines began increasing in the late 20th century, estimates of available power were usually based on kinetic energy flux as in the wind industry [8], and this assumption continued to be used in detailed assessments of tidal resource as the first industrial-scale turbine installations in the UK were under discussion and development [9,2].

With recent advances in tidal stream technology in the following decade, however, more realistic assessments of energy extraction have become necessary. Device designers can model an individual device in detail to develop its structural, hydrodynamic

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and electrical design, but the need to model large arrays of devices in real tidal channels at an appropriate level of complexity poses a significant challenge for the industry at present. Ideally, arrays of tens of turbines would be modelled using three dimensional blade-resolved simulations in domains containing the real bathymetry of a tidal site, with resolution sufficient to capture all wake mixing effects. At present, however, solving such a scenario would have an infeasibly high computational cost, particularly if an iterative solution to find optimal turbine design or siting was required. As such, a variety of different approaches to computational, experimental and theoretical array modelling have been investigated by various authors [26], each making assumptions to allow these large-scale problems to be tractable.

One approach is to use a computational fluid dynamics code which solves the three-dimensional Reynolds-Averaged Navier-Stokes (RANS) equations, but to simplify the channel geometry and the turbine model; this often involves a rectangular channel and use of actuator models for the turbines, such as in Refs. [3,14]. It is also possible to simplify the turbine model using Blade Element Momentum (BEM) theory, which [17] utilised to study the performance of staggered turbines in an array. However, the use of three-dimensional modelling of any kind significantly limits the number of devices which can be considered due to computational cost with expanding domain size. Experimental modelling of arrays has been similarly restricted to studying the interaction of small numbers of devices at the largest possible Reynolds numbers achievable in existing facilities: for example [23], studied a single row array of up to five scale rotors, while [19] considered three porous discs in a staggered row arrangement and [18] reported results for two axially aligned turbines, one behind the other.

In order to computationally study large scale resources with arrays of many tens of devices, it is necessary to significantly reduce the complexity of the problem. This can be done by using simplified numerical models in which individual turbines or arrays are represented as either enhanced friction terms or momentum sinks within a simulation of the two-dimensional shallow water equations [6,28]. If sufficiently simplified, these models can be computationally efficient enough to allow optimisation of array layouts [5,10]. It should be noted, however, that these models, being two-dimensional in nature, cannot capture the complete three-dimensional wake mixing effects, and as such are necessarily limited in their ability to simulate arrays with closely-spaced turbines where these effects are important, unless specifically accounted for through sub-grid models [22,27].

In order to evaluate the outputs of resource modelling, it is necessary to understand the theoretical limits of energy extraction by arrays of devices within tidal channels, just as the Lanchester-Betz limit is used as a theoretical benchmark in wind turbine design. The simplest influence of channel blockage was accounted for in the theory model of [12]; complementing their previous analytical model of available tidal power from the head-driven flow through a tidal channel [11]. Later models such as that of [20] and [7] have extended this framework with additional degrees of complexity to allow consideration of arrays of multiple devices. This theoretical framework, focusing on simplified models of fluid flow to obtain maximum limits of energy extraction, has been developed over recent years as described in the following paragraphs.

[11] examined the energy balance of the flow through a tidal channel in order to predict the power available for extraction by tidal turbines. It was discovered that the maximum power available to be extracted by any device within it is not directly related to the kinetic flux through the channel, because of the constrained nature of the flow. Instead, by considering momentum balance within a constrained tidal channel, they developed an expression for maximum average power, P_{max}, available in terms of the tidal head, a, developed along a channel which has an undisturbed volumetric flux of Q_{max}:

$$P_{max} = \gamma \rho g a Q_{max} \tag{1}$$

where ρ is the density of sea water, g is acceleration due to gravity, and γ is in the range 0.2–0.24. This range of γ reflects uncertainty in the model regarding the appropriate drag law, contributions from bed friction and channel exit effects, etc. This derivation considered the energy balance between the potential energy of the tidal head and the flow resistance due to the turbine and other effects such as bed friction. Their study assumed that the thrust of the tidal turbine was equally spread across the entire channel bed, i.e. that a turbine array completely filled the channel at one point along its length. In real channels, this is clearly not practical, and so Garrett and Cummins extended their work further by considering Linear Momentum Actuator Disc Theory (LMADT) to model flows through and around a single turbine in a constant mass flux channel [12].

Given that, in a real tidal channel, it is likely that tidal turbine arrays will only occupy a proportion of the channel's cross-section, due to uneven bathymetry, turbine geometry, requirements for shipping channels and the like, this work has been further developed by many others, in different ways. Vennell combined the single turbine model with the variable channel flow model [24] and showed that for a homogeneous turbine fence completely occupying the width of the channel, the optimal turbine through-flow varies depending on the proportion of the channel cross-sectional area filled by turbines. Highly blocked channels achieve maximum power output with greater flow induction through each turbine. A following study also showed that more power can be achieved by increasing the number of turbines in a single row than by creating additional rows of turbines [25].

The single turbine model has several limiting assumptions, some of which were addressed by Refs. [29,13] to allow more complex cases to be considered, in particular the case of open channel flow with a non-zero Froude number. This case is of particular interest where high channel blockage with a large number of turbines creates a significant flow obstruction and causes considerable head loss across the turbine fence.

The work of [20] extended the single turbine model of [12] in a different direction to enable turbine fences of finite length, which do not completely occupy the width of the channel, to be modelled. Their work introduced the idea of scale separation to allow an entire row array to be modelled as one device in a channel, while all the turbines within the array are similarly modelled as devices within a separate array channel defined by the array core flow properties. This model maintained the assumptions of constant mass flux and constrained (rigid lid) flow of [12]; while introducing an additional assumption of scale separation between device scale and array scale wake mixing. This model predicted a maximum power coefficient for an infinitely wide channel of 0.798, a significant increase on the completely unconstrained Betz limit of 0.593. The assumption of complete scale separation was partially addressed in later work [21] to include short row arrays with significant device wake expansion, and simultaneously validated with three-dimensional flow analysis which showed better agreement with partial row array model results than it did with the original single turbine [12] model.

The present paper will extend the model of Nishino and Willden in a different direction, proposing a further level of scale separation to investigate the potential for splitting a long row array of turbines into smaller sub-arrays, still arranged in a row. As such, a review of previous LMADT modelling will first be provided, followed by the details of the new sub-array model. The results obtained from this