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Fast optimisation of tidal stream turbine positions for power generation in small arrays with low blockage based on superposition of self-similar far-wake velocity deficit profiles

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ABSTRACT

Far wake velocities of a single horizontal axis three-bladed turbine in shallow flow have been measured previously in the laboratory and shown to have self-similar velocity deficit profiles. Wake velocities of arrays of turbines with one, two and three transverse rows have also been measured and simply superimposing the velocity deficits for a single turbine is shown to give accurate prediction of combined wake width and velocity deficit, accounting for variable downstream blockage through volume flux conservation. Array efficiency is defined as the ratio of total power generated to what would be generated by the same turbines in isolation. From prescribed initial turbine positions, generally determined intuitively or by practical considerations, adjusting the turbine positions to increase the power from each turbine, using the chain rule, shows that relatively small movements of 3–4 rotor diameters may increase array efficiency to over 90%.

1. Introduction

Several prototype tidal stream turbines have been developed and deployed individually showing good performance. The next stage is to deploy as arrays for significant energy capture and at least two sites are planned for deployment within the next decade. Array interaction effects due to wake velocity deficits that reduce power of downstream turbines are clearly important particularly since power for a given power coefficient is proportional to velocity cubed. Models of flow in arrays have been based simply on the idea that a turbine thrust in a shallow water depth-averaged model may be imposed to simulate wake characteristics, e.g. Refs. [3,5,13]. However comparison with experimental data for a fence of turbines close to a headland has been shown to underestimate velocity deficit [4]. This has been supported through some investigations in parallel channel flow by the authors (unpublished) using the depth-averaged model of [18]. With an axial induction factor adjusted to give the correct thrust for a particular mesh, wake velocity deficits were considerably underestimated compared with experiments presented herein. Artificially increasing thrust coefficient could improve the wake velocity locally but the downstream variation was not correct and wakes widths were invariably too narrow. This approach had previously also been applied to arrays of pile groups where it was shown that large-scale wake features may be reproduced by increasing drag coefficients from their physical values [2]. Ref. [5] optimised power generation from arrays by moving turbine positions using a gradient based algorithm with the adjoint approach.

Wake interaction effects may also be investigated using computational fluid dynamics (CFD). Blade element momentum (BEM) methods coupled with Reynolds averaged Navier Stokes (RANS) models provide a computationally tractable approach for small turbine arrays. Ref. [8] used this approach for up to 14 turbines with some manual optimisation based on observations for improving power generation from three-turbine arrays. This RANS BEM approach has since been compared with experiment for array configurations presented in this paper [10].

Here we are concerned with general arrays with low blockage and low Froude number. Free surface effects will be minimal. Experimental measurements of wake velocity are available for a single turbine and arrays with one, two and three rows. The velocity deficit in the far wake of a single turbine shows two-dimensional self-similar characteristics [15]. For multiple rows

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wake velocities will be compared with those obtained by superimposing the velocity deficit of a single turbine to account for the velocity reduction of one turbine in the wake of another. This approach has been applied to the self-similar flow fields of wind turbine wakes [6]. Recently [7] compared this approach with three others for flow through an array of two turbines computed using LES (large eddy simulation) and showed it gave better predictions than wake merging methods which have also been applied to tidal stream turbine arrays, neglecting blockage [11]. Using the velocity deficit superposition approach, positions of turbines will be moved from prescribed initial positions to increase individual power generation and hence total power using an algorithm based on the chain rule.

2. Experimental arrangement

The experimental arrangement has been described previously for studies of the flow downstream of a single horizontal axis three-bladed rotor, a transverse row of these rotors and the arrays of this study in shallow turbulent flow [10,14,15] respectively. This is summarised here. Velocity measurements were made with Nortek ADVs, forces with a strain-gauged load balance and power from torque supplied by a DC motor (with friction subtracted) times the rotation speed measured by a digital encoder; details are reported in Refs. [14,15]. The rotors had diameter $D = 0.27$ m in a channel of width $w = 18.5D$ (5 m) and depth $h = 1.67\ D$ (0.45 m). The average flow velocity was 0.46 m/s. For each array configuration a central upstream turbine axis was located 22D from the inflow, at mid-span and at mid-depth. The foil sections were selected for high lift to drag ratio at a chord Reynolds number of approximately 3×10^4 (typical at three-quarter radius at a tip speed ratio of 4.5) and with radial variation of pitch angle and chord length selected to represent the operating point of a full-scale rotor [19]. Streamwise thrust, applied torque and rotational speed of each rotor were sampled at 200 Hz for each rotor. Measured force is reduced by the drag measured on the supporting tower to give thrust. Measurement of the mean flow and turbulence characteristics taken at the plane of the upstream row indicate that the vertical profile of mean velocity follows the log law. Depth average turbulence intensity is 12% in the streamwise direction and 9% in the vertical and lateral directions. The integral length scales of the ambient turbulence measured by a two point cross correlation method at mid-depth are 0.56h, 0.33h and 0.25h in the streamwise, transverse and vertical axes respectively. Sample duration was 900 s for these measurements. Length-scales were also estimated by an auto-correlation method providing similar values at mid-depth. It is well known that horizontal scales are greater than vertical in shallow flows and these scales are of similar magnitude to field measurements, e.g. Ref. [12]. Experimental measurements for this paper were obtained for rotors arranged in six different array configurations with three to twelve turbines. For each rotor constant retarding torque was applied by the dynamometer system and defined to develop a tip-speed-ratio of 4.5 when in isolation. For each array a number of wake traverses were obtained at planes downstream of the final row of the array. These included vertical profiles directly downstream of each rotor and transverse profiles at hub-height. Each wake traverse comprised samples of 60 s duration sampled at 200 Hz. During each wake traverse, streamwise force, torque and rotational speed for each rotor were recorded.

3. Self-similar velocity deficit superposition and blockage

For the wake of the single turbine of this study the velocity deficit has been shown to become two-dimensional and self-similar for distances greater than 8 diameters downstream [15]. At maximum power coefficient C_P with $\beta = 4.5$ the equations for centreline velocity deficit ΔU_{max} and wake half width y_{half} at downstream distance x are given by

$$\frac{\Delta U_{max}}{U_0} = -0.126 + 0.8639 \Big/ \sqrt{x/D} \tag{1}$$

$$\frac{y_{half}}{R} = 0.5 + 0.4118\ \sqrt{x/D} \tag{2}$$

with the transverse deficit $\Delta U(y)$, where y is distance from centreline, given by

$$\frac{\Delta U(y)}{\Delta U_{max}} = exp\left(-\ln(2)\frac{y^2}{y_{half}^2} \right) \tag{3}$$

These formulae have also been shown to give remarkably accurate prediction of depth-averaged velocity deficit extended to distances between 4D and 8D downstream. The torque in the array was controlled to give the constant value associated with maximum power coefficient C_P at $\beta = 4.5$ for ambient flow conditions, giving $C_T = 0.89$. The β values for downstream turbines defined relative to the ambient flow varied in the range of about 4–6. The corresponding thrust coefficient of downstream turbines varied in the range 0.74–0.9; this is the dominant factor in determining wake characteristics and the same velocity deficit formulae are assumed to apply.

To implement wake superposition first the velocity deficits of upstream turbines are imposed on the flow field, represented on a Cartesian mesh. The velocity at downstream turbines will be reduced if within the wake of an upstream turbine thus defining a new onset velocity. This velocity may be interpreted as the average over the disc area but this is within 1% of the hub velocity for the cases studied. The velocity deficits of these downstream turbines are then superimposed on the flow field. In addition blockage needs to be considered for comparison with experiment in a confined channel. We consider three rows (row 1 upstream to row 3 downstream). First assume that the onset velocity U_0 applies at row 1 with upstream volume flux q_0. This gives the downstream flux q_1 at row 2 with superposition of the velocity deficits from row 1 turbines. q_1 will be less than q_0 and this is corrected by a blockage correction factor q_0/q_1 so that the velocity onset on the turbines is $U_0\ q_0/q_1$. The velocity field downstream of row 1 is thus defined by this velocity with superimposed velocity deficits and stored on a mesh. This provides a first approximation for the onset flow for row 2 and with superimposed velocity deficits velocities at row 3 are defined giving a flux q_2 at row 3. q_2 is less than q_0 and the velocity onset on row 2 is multiplied by a blockage correction factor q_0/q_2 to give the correct flux at row 3. The velocity field downstream of row 2 is thus modified and the velocities stored on the mesh updated. This now provides the first approximation for the onset flow for row 3 and with superimposed velocity deficits velocities downstream (either 4D or 8D here) are defined giving a flux q_3. Again this will be less that q_0 and the velocity onset on row 3 is multiplied by a blockage correction factor q_0/q_3 to give the correct flux. Further rows may be incorporated in the same way but are not considered here.

This is for one flow direction and the flow field is steady. In tidal flows the flow reverses and the wake interaction process is applied over rows in the reverse order. In the code velocity direction is defined by the angle of incidence so variable angle onset flow may be taken into account although only values of 0 and 180^0 are investigated here. This does imply that the residual wake from one half cycle has no effect in the following half cycle.

Experimental results for 6 array configurations in uni-