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# On a universal model for the prediction of the daily global solar radiation

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A model to predict the mean expected daily global solar radiation, H(n) on a day n, at a site with latitude φ is proposed. The model is based on two cosine functions. A regression analysis taking into account the mean measured values $H_{m.meas}(n)$ obtained from SoDa database for 42 sites in the Northern Hemisphere resulted in a set of mathematical expressions of split form to predict H(n). The parameters of the two cosine model for $0° < \varphi < 23°$ are obtained by regression analysis using a sum of 3–8 Gaussian functions, while for $23° < \varphi < 71°$ the two cosine model parameters are expressed by a sum of exponential functions or the product of an exponential and a cosine function. The main equation of the model and the set of parametric expressions provide H(n) for any φ on Earth. Validation results of this model are provided along with the statistical estimators NMBE, NRMSE and t-statistic in comparison to the corresponding values from three databases of NASA, SoDa and the measured values from ground stations provided in Meteonorm.



## 1. Introduction

The mean expected daily global solar radiation, H(n), on the horizontal plane in any place and on any day is an important factor and it may serve as data input in sizing projects related to solar collector and PV systems, as well as in meteorological projects. Therefore, solar radiation data collection is carefully managed and elaborated in any country. Many papers have been published outlining models which provide H(n) estimates. A couple of those models like the Iqbal model C and the ASHRAE [1–3] are semi-empirical and predict the beam and diffuse components of the global solar radiation in a site leading to an easy determination of the global daily values. Both are based on the theoretical and experimental estimation for the site concerned of certain physico-chemical quantities, optical properties of the solar light attenuation in the atmosphere, and simulation of the processes, even including multiple reflection processes between ground and sky. Other models starting from the Ångström-Prescott model [4] provide the H(n) values in any place based on various empirical expressions with the monthly mean daily fraction of possible sunshine hours [5–10]. An analytic approach is presented in the meteorological radiation models [11,12]. Another family of models correlates H(n) with ambient temperature, humidity, cloudiness, associated with the clearness index, and other meteorological parameters [13–16], reaching up to models using artificial intelligence [17], while a third group of models provides expressions how to determine H(n) in a site with parameter the day of the year [18–23]. The regression analysis is the general tool to determine the values of the parameters through which these models are described. These values are valid for the region the model is tested, i.e. the latitude, and longitude and the microclimate, in general. The papers that have been published, as the abovementioned ones, present the mathematical expressions of the proposed models for the specific regions and provide an elaboration of the values of the parameters they depend on. The H(n) model expressions are grouped according to:

1. the day of the year, n, or some more complex expressions based on cyclic functions [20–22]. This model holds for $25° < \varphi < 60°$.

$$H(n) = A + B\cos\left(\frac{2\pi}{365}n + C\right) \quad (1)$$

2. the actual sunshine hours on a day, S, over the theoretical daylight hours on that day, $S_o$ based on the Ångström-Prescott model and its evolution with more complex functions [5–12].

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$$\frac{H}{H_{ext}} = a + b\left(\frac{S}{S_o}\right) \tag{2}$$

where $H_{ext}$ is the daily extraterrestial solar radiation on the horizontal plane [1].

3. several mixed-type expressions as below [13,22,23].

$$\frac{H}{H_{ext}} = a + \sum f\left(\frac{S}{S_o}\right) + \sum f'\left(\frac{S}{S_o}\right) + .... + \sum f(T_{max}^{m}) + \sum f(RH) \tag{3}$$

where $T_{max}$ is the maximum ambient temperature of the day and RH the relative humidity of the same day.

A, B, C, a, b etc. are parameters to be determined for any site by regression analysis. Based on the analysis carried out in the present work the least number of H(n) values required for a well-correlated fitting in a function as that in Eq. (1) in order to obtain A,B,C is 6. Eq. (1) holds for latitudes around 23°-60° and provides H(n) values with a very good coefficient of determination, $R^2$, around 0.97–0.99 [24]. Mean monthly daily values of the global solar radiation may also be used for the need of the fitting as these values are close to the solar radiation value of the representative day of the month. Mean monthly daily values are provided by many databases like PVGIS, SoDa, Meteonorm, PVWatts, NREL, NASA, RETScreen [25–31]. Having determined H(n) the hourly global solar radiation for the site can be determined by the models outlined in [20,32–35]. Nevertheless, it is preferable that a universal model be set up to provide H(n) for any day at any site, without the need of any database, instead of performing regression analysis for each region to determine the model parameters.

## 2. Model outline

The present investigation proposes a universal model, which predicts the global horizontal solar radiation H(n) as a function of the day (n) of the year, provided in Eq. (4), along with a set of parametric mathematical expressions, which depend on the latitude φ. This is a two-cosine model applicable both in the Northern and Southern Hemispheres. A regression analysis was applied to the global solar radiation data from 42 sites from 0°N (Equator) to 71°N, as shown in Table 1, obtained from the SoDa database [26]. The mathematical expression for the proposed model is given below:

$$H(n) = A_1 + B_1 cos\left(C_1\frac{2\pi}{365}n + D_1\right) + B_2 cos\left(C_2\frac{2\pi}{365}n + D_2\right) \tag{4}$$

The regression analysis of the 12 mean monthly daily global horizontal solar radiation values taken from the SoDa database and carried out for each one of the 42 sites from Eq. (4) gave the values of the unknown parameters $A_1$, $B_1$, $B_2$, $C_1$, $C_2$, $D_1$ and $D_2$.

The fitting results of the model using the regression analysis in MATLAB, is shown in Figs. 1–4, for various sites, along with the coefficient of determination $R^2$, whose value for any latitude and longitude was in the range 0.97–0.99, and the Root Mean Square Error (RMSE) whose value in the range 0.09–0.34 kW h $m^{-2}$ $d^{-1}$.

The methodology for determining the unknown parameters of the model in Eq. (4) is presented in the following section.

## 3. Mathematical expressions and parameterization of the model

As said, the regression analysis done for the H(n) data of the 42 sites provided a set of values for the seven parameters mentioned above. Each of those seven parameters has been fitted separately to a split function of φ. The split functions consist of two parts; the first part holds for the sites with 0°< φ ≤ 23.6°N and the second part of 21°< φ < 71°N. The second part was started from 21°N in order to bridge the region around φ = 21°-23.6° and result to a smooth continuous fitting taking into account those two different fitting functions. This approach provides a better prediction for H(n) in the transition geographical region.

The required general expressions for the parameters $A_1$, $B_1$, $B_2$, $C_1$, $C_2$, $D_1$ and $D_2$ are given in Eqs. (5)–(11) and are based for the first part (tropical region) with 0° < φ ≤ 21°N on a series of three to eight Gaussian functions with φ (in degrees) as argument, while the second part which represents regions with 21°< φ < 71°N is composed by a series of exponential functions. In the case of the parameters $B_1$ and $B_2$, the second part of the split function for 21°<φ < 71° is expressed by the product of an exponential term with a cosine function. In the transition region, 21°< φ ≤ 23.6°N, as earlier mentioned, the parameters are giving better results on taking the average of both functions. The values of the parameters $a_i$, $b_i$, $c_i$, which appear in the Gaussian functions, are given in Table 2. The regression analysis followed determined the number of Gaussian terms which provided the best fit. The values of $a_i$, $b_i$ and $c_i$, differ for each parameter, $A_1$, $B_1$, $B_2$, $C_1$, $C_2$, $D_1$, $D_2$. The fitting results for the parameters $A_1$, $B_1$, $B_2$, $C_1$, $C_2$, $D_1$, $D_2$ along with their coefficient of determination $R^2$ are shown in Figs. 5–11

$$A_1 = \begin{cases} \sum_{i=1}^{3} a_i\, exp\left[-\left(\frac{\varphi - b_i}{c_i}\right)^2\right], & 0^o < \varphi \le 23.6^\circ N \\ 12.680\cdot\exp\left(-1.523\cdot\varphi\frac{\pi}{180}\right) - 15.820\cdot\exp\left(-5.918\cdot\varphi\frac{\pi}{180}\right), & 21^o < \varphi < 71^\circ N \end{cases} \tag{5}$$

$$B_1 = \begin{cases} \sum_{i=1}^{8} a_i\, exp\left[-\left(\frac{\varphi - b_i}{c_i}\right)^2\right], & 0^o < \varphi \le 23.6^\circ N \\ -5.336\cdot\exp\left(-1.270\cdot\varphi\frac{\pi}{180}\right)\cdot\cos\left(1.373\cdot\varphi\frac{\pi}{180} - 1.795\right), & 21^o < \varphi < 71^\circ N \end{cases} \tag{6}$$