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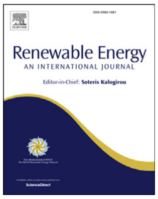

Effect of free surface deformation on the extractable power of a finite width turbine array

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ABSTRACT

The effect of free surface deformation on the power extracted by a tidal turbine array partially spanning a wide channel is investigated using a theoretical model. Two predominant flow scales are assumed; turbine-scale flow, and array-scale flow, which are analysed as quasi-inviscid open channel flow problems in which conservation of mass, momentum, and energy are considered, and coupled through kinematic and dynamic boundary conditions. Power extraction may be maximised by determining the optimum inter-turbine spacing, which also enhances efficiency (ratio of power generated to power removed from the flow). Power extraction and efficiency increase as Froude number increases, improving open channel array performance. In the infinitely wide channel limit, the extracted power depends only on Froude number and local blockage (ratio of turbine to local flow passage areas). At zero Froude number, the peak power coefficient increases from the Lanchester-Betz limit (0.593) to 0.798, occurring when the local blockage ratio is approximately 0.4. Froude numbers in the range of 0.1–0.2, typical of prospective tidal energy sites, increase the peak power coefficient by an additional 1%–4.5% when the array occupies a negligible fraction of the channel, increasing further as a greater proportion of the channel is occupied.

It is well recognised that the power extracted by a turbine in a channel depends on the blockage ratio, *B*, the ratio of total turbine frontal area to channel cross section. Garrett and Cummins [6] used a one-dimensional model to show that, for a single turbine in a channel with a rigid lid, the maximum power extraction was proportional to $(1-B)^{-2}$. Such a simple relationship was not derived in models incorporating a free-surface, but it was still found that power extraction could be maximised by blocking a large fraction of the channel cross-section [11]. However, this may not be possible for various environmental, technical, and regulatory reasons, and does not account for the reduction of the flow for a given head difference across the channel that occurs as a consequence of the increased resistance imposed on the flow by the turbines. Garrett and Cummins [5] and later Vennell [9] investigated the response of a channel to tidal energy extraction, showing that the flux through the channel reduces when the hydrodynamic drag of the turbines becomes significant relative to the resistance in the undisturbed channel, reducing the energy extracted by the turbines.

It may not be feasible to deploy turbines to span the entire width of the channel due to various economic and regulatory constraints in addition to bathymetric variations and the need to allow passage for shipping and marine life. If the turbine array does not span the entire channel width, then there exist two principal flow dynamics in the channel: the first being flow phenomena that occur close to each tidal turbine; and the second being much larger phenomena that occur on the scale of the array width. The acceleration of the flow in the array bypass due to the aggregate thrust applied by the tidal turbines and corresponding reduction in flow speed through the array results in a different estimate of the power that may be extracted by the turbines as compared to analyses in which the turbines span the entire channel width.

Nishino and Willden [7] introduced the concept of scale separation in which array-scale flow phenomena occur more slowly and over longer distances than turbine-scale phenomena, allowing the array-scale and turbine-scale flows to be treated as two loosely coupled problems. This allowed the application of the rigid lid model for an array of tidal turbines partially spanning a wide channel. The array-scale problem provided boundary conditions for the turbine-scale model, which was used in turn to determine the thrust applied to the flow, and thus the dynamic coupling to the

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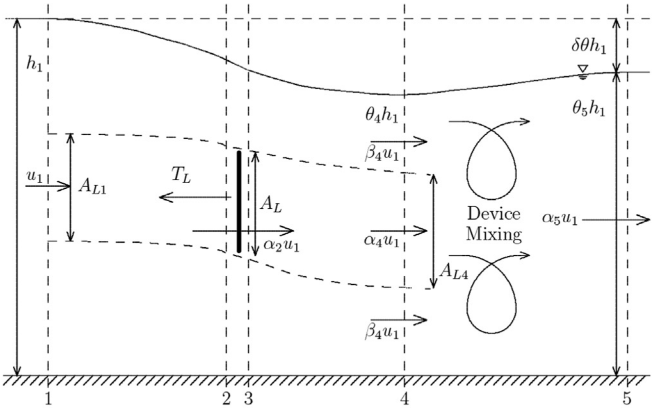
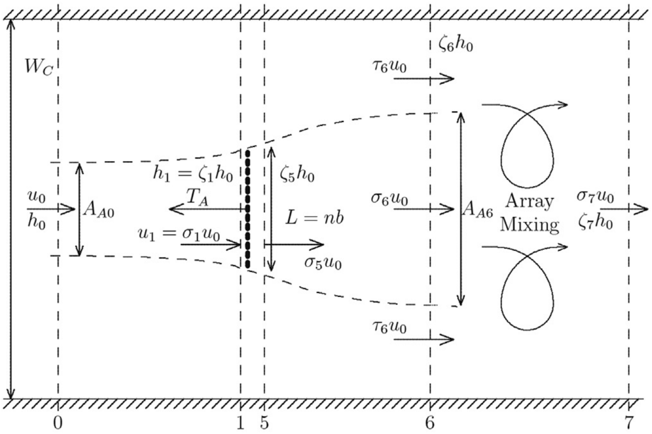
array scale problem, and the power extraction.

Tidal channels are free surface flows, and as such the water depth must change in order that energy be removed from the flow. The Froude number is typically up to 0.2 in channels of interest for tidal energy development, such as the Pentland Firth or Cook Strait. The rigid lid model assumes that the Froude number is negligible, restricting its application to low Froude number channels and to conditions where the impact of the turbine array on the flow is small, which may not accurately reflect sites of current interest. The reduction in water depth and acceleration of the downstream flow in the turbine (or array) wake will be particularly important when considering multiple rows of turbines. Linear Momentum Actuator Disc Theory (LMADT) has been applied to turbines in constrained flow with a deformable free surface [11] and extended to consider downstream mixing [3] to allow analytic modelling of turbine arrays where free surface deformation may be important and the turbine array spans the full channel width.

In the present work, a general model incorporating a free surface into the scale separation framework is developed to provide an analytic model of a turbine array partially spanning a wide channel. Particular consideration is given to the limiting case of an infinitely wide channel (array blockage tends to zero), as many tidal arrays may only occupy a small fraction of a very much wider tidal channel. The analytic model is derived with respect to an upstream reference velocity, and may be embedded within a larger model, such as that of [5], to account for changes in the mass flow rate due to increased resistance of the flow by the turbines. Note however that the assumption of constant mass flux and upstream Froude number becomes exact in the case of turbines operating in an infinitely wide channel, which may be a suitable approximation for many tidal arrays, removing the necessity for a larger channel model. Furthermore, it is assumed that the flow is inviscid and that no mixing occurs other than in the mixing zones of the two models described below.



(a) Turbine-scale flow mixing and free surface change.



(b) Array-scale flow mixing.

Fig. 1. The scale-separated partial array model: (a) turbine-scale, and (b) plan view of the array-scale.

1. Model

A tidal turbine array of width L, composed of n turbines, each of frontal area A_L, partially spanning a channel of width W_C and depth h_0 is shown in Fig. 1. There is a flow passage of width b encompassing each turbine, such that $L = nb$. Energy is conserved everywhere except for the discontinuity in static head across the turbines and in the mixing zones at the turbine and array scales.

At the turbine-scale (Fig. 1b), the inviscid, incompressible flow at station 1 slows as it approaches the upstream side of the actuator disc at station 2. A step-change in static head occurs in the streamtube encompassing the turbine between stations 2 and 3 as a result of the applied thrust. Downstream of station 3, the flow in the wake of the disc continues to slow until hydrostatic pressure variation is achieved at station 4. The expansion of the streamtube encompassing the actuator disc between stations 1 and 4 results in an acceleration of the flow outside the streamtube, which results in a velocity differential between core and bypass flows at station 4. These velocities necessarily mix between stations 4 and 5 to produce a uniform flow speed at 5, where the pressure variation is again hydrostatic. The turbine-scale model results in an increase in velocity from 1 to 5 and a decrease in static head.

It is assumed that the velocity u_0 at the upstream boundary of the channel is spatially uniform and unaltered by turbine operation. The water depth is permitted to vary along the length of the channel. At the array scale (Fig. 1a), the inviscid, incompressible flow upstream of the turbine array slows, resulting in an increase in static head between stations 0 and 1. A static head and velocity discontinuity is established between stations 1 and 5, as described above, and the streamtube encompassing the array expands as the flow speed reduces until a spanwise uniform hydrostatic pressure variation is achieved across the width of the channel at station 6. The expansion of the streamtube causes acceleration of the flow around the array, resulting in a velocity differential at station 6 between array core and bypass flows, which necessarily remix to yield a spanwise uniform flow speed and hydrostatic pressure variation at the outlet at station 7.

Flow phenomena occur over two principal length scales; the turbine diameter D, and the array length L. Turbine-scale energy extraction and mixing scales on the turbine diameter, whereas the flow around the array scales on the array width. If there are many turbines in the array, then $L \gg D$ and the three-dimensional turbine-scale flow occurs over a much shorter spatial and temporal scale than flow around the array, which will tend to being two-dimensional, as the array width is much greater than the channel depth. The difference in the turbine and the array length scales is assumed to be sufficiently large such that turbine-scale core and bypass flow wake mixing is completed upstream of cross-stream depth equalisation (hydrostatic pressure recovery) in the array scale problem. The partial fence problem may thus be considered in terms of the two loosely coupled quasi-inviscid problems shown in Fig. 1; a turbine-scale problem and an array-scale problem. Each problem is evaluated separately as an open channel flow, adapting the open channel model of [11] to the two-scale partial fence problem.

The turbine-scale and array-scale problems are non-dimensionalised by scaling the velocities and static heads in terms of the upstream height and velocity at station 0 in the array-