Renewable Energy 85 (2016) 215–227

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Integration of cost modelling within the micro-siting design optimisation of tidal turbine arrays

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ARTICLE INFO

Article history:
Received 18 September 2014
Received in revised form
1 June 2015
Accepted 7 June 2015
Available online xxx

Keywords:
Marine renewable energy
Tidal turbines
Gradient-based optimisation
Adjoint method
Shallow water equations
Array layout
Cable-routing
Financial-return

ABSTRACT

The location of individual turbines within a tidal current turbine array – micro-siting – can have a significant impact on the power that the array may extract from the flow. Due to the infancy of the industry and the challenges of exploiting the resource, the economic costs of realising industrial scale tidal current energy projects are significant and should be considered as one of the key drivers of array design. This paper proposes a framework for the automated design of tidal current turbine arrays in which costs over the lifespan of the array may be modelled and considered as part of the design optimisation process. To demonstrate this approach, the cost of sub-sea cabling is incorporated by implementing a cable-routing algorithm alongside an existing gradient-based array optimisation algorithm. Three idealised test scenarios are used to demonstrate the effects of a financial-return optimising design approach as contrasted with a power maximisation approach.

1. Introduction

Tidal current turbines are devices which convert the momentum of tidally induced ocean currents into electricity. Much as with wind power, several individual turbines may be formed into an array to yield power on an industrial scale. Determining the optimum location of turbines within an array – micro-siting – is an issue of critical importance, on which the viability of the project may hinge. Rapid spatial variations in current flow speed can be caused by complex bathymetry and the presence of the turbines themselves. Since the power extraction of a turbine is dependent upon the cube of that flow speed (i.e. is highly sensitive to it), optimisation of the micro-siting design is a complex and challenging problem. Funke et al. [10] have demonstrated in test scenarios that such micro-siting optimisation has the potential to increase the power extracted by an array by 33% as compared to an array of the same number of turbines arranged in a rectangular grid.

Just as power production is dependent on the turbine micro-siting design, so too may certain *costs* be dependent on turbine location. These location-dependent costs, such as the cost of cabling, water depth or difficulty of installation, may have just as great an impact on the project viability and must be considered in the design process. As noted by Thomson et al. [26] there has been a general focus across the renewables industry on design optimisation based solely on energy yield. The goal of this paper is to develop a more holistic approach to array design which balances both energy yield and cost, and thus enables array developers to maximise their overall return on investment.

For this work, sub-marine cabling costs have been chosen as an example location-based cost. In offshore wind projects, connection

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http://dx.doi.org/10.1016/j.renene.2015.06.013

costs typically represent 18–20% of the capital cost of a project [18,22]. Research into the cost of elements of tidal current power installations has suggested that these costs may be similarly significant, and the largest cost affected by micro-siting design [1]. Negative wake interactions between turbines may motivate the spacing of turbines – where lease area allows – far apart from each other and conditions may result in higher flow velocities out in the main channel away from land. However, as spacing between turbines and distance from shore increases, so too does the cost of the cabling required to connect the turbines to the power grid. Thus the requirement to minimise the cost associated with increased cable length will likely be a competing factor to maximising power output.

The contribution of this paper is a novel method of optimising turbine micro-siting design, which integrates evaluation and consideration of both costs and benefits. Cable length minimisation and power-extraction maximisation are integrated and optimised using an efficient gradient-based algorithm. This approach requires fewer iterations than alternate (for example genetic algorithm) approaches meaning that a more computationally expensive (and therefore more accurate/realistic) prediction model may be used.

In the following section, the array micro-siting design optimisation problem is formulated mathematically, and it is shown how the financial return is defined as a function of incomes – such as power extracted by the array, and costs – such as the length of cabling required to connect the turbines. In Section 3, *OpenTidalFarm*, a code developed by Funke et al. [10] is presented as a method to optimise the array layout to maximise the power extraction of the array, using the shallow water equations and a gradient-based optimisation approach. In Section 4 the cable routing problem is outlined, and previous work in tackling it is explored. The problem is mathematically formulated and it is shown that the proposed integration into a gradient-based framework is valid. In Section 5, a model is developed for this application leading to Section 6 in which the challenges of integrating the *OpenTidalFarm* and cable routing models are examined. Finally the approach is demonstrated on idealised test-cases in Section 7.

2. Problem formulation

The overall goal of this work is to optimise the financial return over the array lifespan, R_{fin}, for the developer of an array of n tidal turbines located within a bounded site – the 'turbine area'.

The domain is modelled in two-dimensional Cartesian space and the coordinates of the turbine locations are encoded in a $2n$-long vector, $\boldsymbol{m}$, where

$$\boldsymbol{m} = (x_1, y_1, x_2, y_2, \cdots, x_n, y_n)^T.$$

R_{fin} is considered to be a function of $\boldsymbol{m}$ and is therefore maximised through adjusting the turbine locations,

$$\max_{\boldsymbol{m}} \quad R_{\text{fin}}(\boldsymbol{m}). \tag{1}$$

In order to obtain a framework through which cost and income models can be integrated in a modular fashion, R_{fin} is expressed as the sum of income functions and cost functions. For example (1) may be expressed as

$$\max_{\boldsymbol{m}} \quad I_P(P(\boldsymbol{m})) - C_C(L(\boldsymbol{m})), \tag{2}$$

where $P(\boldsymbol{m})$ is the power extracted from the flow by the turbines, $I_P : \mathbb{R} \to \mathbb{R}$ maps power output to financial income, $L(\boldsymbol{m})$ is the total length of sub-sea cabling required to connect the turbines to base stations on the shore, and $C_C : \mathbb{R} \to \mathbb{R}$ maps cable length to financial cost. Both P and L can implicitly be written as functions of $\boldsymbol{m}$ when it is understood that for a given array configuration ($\boldsymbol{m}$), the evaluation of P will involve the solution of a problem describing the tidal dynamics and evaluation of L involves the solution of a routing optimisation problem. Further details on the functions I_P and C_C used in (2) may be found in Section 6.

Models which evaluate a physical quantity, such as power extracted by the array, or the length of sub-sea cabling required to service it, can thus be included in the composition of (1) if functions can be identified which map those physical quantities into a financial dimension.

In a general setting, there might be additional location-based cost functionals, such as installation depth, which could be incorporated in a similar fashion as the cable cost, but these are not considered here. For this work, these (and all other costs) are assumed to be constant.

3. OpenTidalFarm

An approach to maximising the power production, $P(\boldsymbol{m})$, has been developed by Funke et al. [10]. This method is packaged in the open-source code *OpenTidalFarm* (*opentidalfarm.org*), and will be summarised here for completeness. *OpenTidalFarm* solves an optimisation problem constrained by the shallow water equations:

$$\begin{aligned} \max_{\boldsymbol{m}} \quad & P(\boldsymbol{m}) \\ \text{subject to} \quad & \boldsymbol{b}_l \le \boldsymbol{m} \le \boldsymbol{b}_u \\ & g(\boldsymbol{m}) \le 0. \end{aligned} \tag{3}$$

The bounds $\boldsymbol{b}_l \le \boldsymbol{m} \le \boldsymbol{b}_u$ constrain the turbines to the turbine area (in this case rectangular in shape, for simplicity) and the inequality constraint, $g(\boldsymbol{m}) \le 0$, implements a minimum distance spacing constraint between adjacent turbines. Turbines are modelled as distinct areas of increased friction, and at each optimisation iteration the performance of the turbine layout is evaluated as the power extracted by the turbines,

$$P(\boldsymbol{m}) = \int_{\Omega} \rho c_t(\boldsymbol{m}) \left|\left|\boldsymbol{u}\right|\right|^3 \, \mathrm{d}x, \tag{4}$$

where ρ is the fluid density, $c_t(\boldsymbol{m})$ is the enhanced friction of the parametrised turbines, and $\boldsymbol{u}$ is the velocity of the flow which, along with the free-surface displacement, η, is the solution to the steady-state shallow water equations

$$\begin{aligned} \boldsymbol{u} \cdot \nabla \boldsymbol{u} - \nu \nabla^2 \boldsymbol{u} + g \nabla \eta + \frac{c_b + c_t(\boldsymbol{m})}{H} \parallel \boldsymbol{u} \parallel \boldsymbol{u} = 0, \\ \nabla \cdot (H \boldsymbol{u}) = 0. \end{aligned} \tag{5}$$

Here, ν is the viscosity coefficient, g is the acceleration due to gravity, H is the total water depth and c_b represents a constant background bottom friction. Note that $P(\boldsymbol{m})$ is a function of $\boldsymbol{m}$ both directly through $c_t(\boldsymbol{m})$ and through the solution of the shallow water equations, $\boldsymbol{u}(\boldsymbol{m})$.

In this paper we consider only the steady-state shallow water model, however this can be generalised to the unsteady case. As such, energy yields quoted in this paper are instantaneous values. This is clearly a weakness in the methodology, however it is useful in two respects. Firstly, using the steady-state shallow water model, proof of concept of integration with the cable routing is demonstrated at a much reduced computational cost as compared to using the unsteady model. Secondly, with flow coming from just one direction, it is much more easy to interpret the arrangement of the turbines and intuitively grasp how the optimal turbine