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Analysis of point-focused, non-imaging Fresnel lenses' concentration profile and manufacture parameters

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ABSTRACT

The purpose of this study is to develop a mechanism of a curve-based, point-focused Fresnel lens concentrator system and use it to examine each spectral segment's distribution patterns on the lens' focal area. The mechanism incorporates optical geometry and ray tracing technique with the components of solar spectrum, refractive index information of lens materials, and the formulation for quantifying the concentrator systems' transmittance loss and prism-tip dispersion loss. In addition to the facet angles' role in refracting the incoming radiation, this research has addressed spectrum-filtering role of the side angle, the angle between the entry face and the side face of each facet on the lens. The theoretical aspect has been elaborated on the basis of the lens' design wavelength. A computerized model has been developed and the simulated outcome compared to the measured data from a previous research. The result of this study provides the information of illumination patterns under a circular lens, which will help to match up various spectrum distributions to their suitable solar applications.

1. Introduction

Fresnel lenses have been among the best choices for solar energy application. They can be compact, light-weighted, low cost, and effective in concentrating solar energy when designed appropriately with right materials such as Polymethylmethacrylate (PMMA). PMMA is a clear, stable, and light-weighted material with optical characteristics similar to that of glass [1]. Besides good resistance to sunlight, PMMA's transmissivity matches the solar spectrum and is, therefore, suitable for Fresnel lens production.

A number of Fresnel lens milestone researches were conducted in 1970's [2–6]. Several design concepts have been studied and recommended, with some of them proceeded into production. US Department of Energy funded a linear Fresnel technology study and recognized the technology commercially viable [7].

Two classes of Fresnel lenses that have evolved include imaging lenses, which focus on one focal point to create accurate images; and non-imaging lenses, which are not intended for photographic accuracy. Non-imaging lenses are more widely used for solar energy applications [8], which requires flexible designs that can cope with solar disk, spectrum, and tracking errors to form uniform flux.

A point focused Fresnel lens, constructed with an array of concentric circular sections of triangular prisms can be viewed as a set of prisms arranged in an annular manner with a slightly curved center and steeper prisms toward the rim. Fig. 1 demonstrates an elliptical-based lens' geometrical relationship with its corresponding circle. As displayed in the figure, spectrum segments of different wavelengths spreads and results in chromatic aberration when passing through a prism.

Fig. 2 displays the illustration of two adjacent facets with an incidence entering the entry face (AB), going through the prism, and exiting at the exit face (AC). As φ'_i increases, the beam tends to spread wider and becomes easier to miss the designated target. As a result, the performances of the lenses with greater φ'_i s are usually not as good as those with smaller φ'_i s. Fortunately, chromatic aberration tends to drop as more refraction occurs at the entry face and less at the exiting face. A curve-based non-imaging Fresnel lenses, which has shorter focal length, serves just the purpose. In addition, curve-based lens offers more design freedom as its first face curvature can be a design parameter that helps to cut the lens' dependency on greater φ'_i.

Minimum deviation condition of a given prism occurs when $\varphi_i = \varphi'_r$. In such condition, the prism achieves maximum transmittance. In addition, small changes in the facet angles (θ) and facet

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Nomenclature

γ	turning angle, the angle between the incidence and the refracted ray
Δs_m	pitch size of facet **m**
η	side angle, the angle between entry face AB and side face BC
η_{avg}	average value of η_{max} and η_{min}
η_{max}	upper bound of side angle to avoid side dispersion
η_{min}	lower bound of side angle to avoid total internal reflection and groove peak dispersion
η_{sd}	maximal side angle to avoid side dispersion
η_{tir}	minimal side angle to avoid total internal reflection
θ	facet angle, the angle between entry face AB and exit face AC
θ_v	facet angle less incident angle of incoming radiation entering entry face AB
λ_d	design wavelength of the lens
τ	tip angle, angle between side face BC and exit face AC
φ_i	incident angle of incoming radiation entering entry face AB
φ_r	refraction angle of incoming radiation leaving entry face AB
φ'_i	refracted ray's incident angle when hitting exit face AC
φ'_r	refracted ray's refraction angle when leaving exit face AC
φ_{mn_i}	incident angle of wavelength segment **n** at entry face of facet **m**
φ'_{mn_i}	incident angle of wavelength segment **n** at exit face of facet **m**
φ_{mn_r}	refraction angle of wavelength segment **n** at entry face of facet **m**
φ'_{mn_r}	refraction angle of wavelength segment **n** at exit face of facet **m**
ψ	complementary angle of half AOV (angle of view)
a	half major axis of the elliptical based lens
b	half minor axis of the elliptical based lens
FR	focal ratio (also called *f number*)
d	prism peak radius
L_{misc}	miscellaneous transmittance loss (combined transmittance loss other than L_{td}, L_{tir}, *and* L_{sd})
L_{td}	transmittance loss due to prism tip dispersion
L_{tir}	transmittance loss due to total internal reflection
L_{sd}	transmittance loss due side dispersion
M	total number of facets on the lens
N	*total* number of wavelength segments
$\tilde{n}_d$	design index of the lens
$\tilde{n}_{inf}$	refractive index of infrared segment through the lens material
$\tilde{n}_l$	refractive index of longest wavelength segment through the lens material
$\tilde{n}_n$	refractive index of nth wavelength segment through the lens material
$\tilde{n}_s$	refractive index of shortest wavelength segment through the lens material
$\tilde{n}_{uv}$	refractive index of UV segment through the lens material
t_m	facet thickness
T_d	tip dispersion loss
T_{mn}	transmittance coefficient of wavelength segment **n** through facet **m**
T_{mn_1}	entry face transmittance factor
T_{mn_2}	exit face transmittance factor
W_{mn}	energy carried by wavelength segment **n** of incident ray through facet **m**
x	focal length of the lens
y	half lens aperture

positioning post only minimum effect on the incidence's turning angle (**γ**). This principle has long been used in prism spectrometers design. One of elliptical shapes closely replicates the curvature required to achieve maximum transmission [9–11]. Elliptical-based Fresnel lenses of that curvature allow the errors in the systems to be minimized and the focusing capability maximized.

Based on the consideration given above, Yeh [12] derived a complete set of ray-tracing equations for elliptical-based, line-focused Fresnel lenses using ray tracing optical geometry. The set of equations, which was built into a computerized model, integrated solar spectrum with refractive indices of PMMA to form different color combinations on the target. That model has been used to study specific spectral segments' distribution patterns on the focal plane of the Fresnel lenses categorized for medium concentration below a hundred suns. Having helped to improve solar spectrum manipulation, the model serves as the foundation of this study, which aims to formulate a point-focused system suitable for applications that call for high concentration of hundreds of suns.

2. Transmittance and losses

This study concentrates on the lens design parameters that include major axis, minor axis, the lens' angle of view (AOV), along with desired design wavelength (λ_d) to calculate the lens specifications that include focal length, aperture, facet pitch, facet count, and the magnitudes of each facet's three angles. The three angles of a facet, namely, the facet angle (θ), the side angle (η), and the tip angle (τ), are displayed in Fig. 2. Facet angle, θ, governs the light bending capacity of the facet and therefore is the most significant angular parameter. The adjustment of the side angle, η, helps to modify the spectrum distribution in the lens' target area.

Applying Snell's Law to an incident ray that enters face AB and exits at face AC to obtain a refraction angle at each face (Fig. 2):

$$\varphi_r = \sin^{-1}(\sin\varphi_i/\tilde{n}_n) \quad (1)$$

$$\varphi'_r = \sin^{-1}(\tilde{n}_n \sin\varphi'_i) \quad (2)$$

The turning angle of the incidence, γ, and η can also be derived from Fig. 2:

$$\gamma = \varphi'_r - \theta_v > \varphi_i - \varphi_r \quad (3)$$

$$\eta = (90-\theta) + \theta_v + (\varphi_i - \varphi_{r_{uv}}) < (90-\theta) + \theta_v + (\varphi'_r - \theta_v) \quad (4)$$

An entry face refraction angle (φ_r) greater than $\sin^{-1}(1/\tilde{n})$ can result in total internal reflection (TIR), which keeps the flux from penetrating the prism. A greater η makes the incidence less apt to TIR, which in turn, allows more light to transmit through the exit face.

Fig. 3 displays an enlarged illustration of the light paths at the intersection of two facets to demonstrate the loss caused by TIR. As illustrated, refracted beam from $\mathbf{I_2}$ is dispersed internally by the side face DC when the side angle η is sharper than a critical angle, η_{tir}, where: