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Transfer functions of solar heating systems for dynamic analysis and control design

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ABSTRACT

Mathematical modelling is the theoretically established tool for developing solar heating systems, e.g. with using transfer functions. If we know the transfer functions of the system, the outlet temperature can be predicted as a function of the input variables (solar irradiance, inlet temperature, environment temperatures), dynamic analysis can be carried out, and stable system control can be effectively designed based on the well-tried methods of control engineering. For these purposes, new, validated transfer functions for solar heating systems are worked out in this study based on a mathematical model, which can be found in the literature and has been applied successfully in the field. The transfer functions are used for dynamic analysis and control design of solar heating systems. The dynamic analysis is presented and the efficiency of the proposed stable control is demonstrated with respect to a real solar heating system.

1. Introduction

Mathematical modelling is the theoretically established tool for developing solar heating systems, e.g. with using transfer functions.

Ordinary differential equation (ODE) models are widely used as they are relatively simple and easy to handle. Among the current collector models, the nonlinear ODE model proposed by Perers and Bales [1] may be the most widely used one, which is the improved version of the quasi-dynamic model from the standard EN 12975 [2].

If there is an external heat exchanger in the system, it can be modelled with the well-known effectiveness-NTU method [3], or the separate sides of the heat exchanger can be assumed to have homogeneous temperatures and can be modelled with ODEs [4].

The pipes of the system may be modelled with ODEs (assuming homogeneous pipe temperatures), or partial differential equations (PDEs) (with the one-dimensional linear heat transfer PDE) [5,6].

Solar storages can be also modelled with ODEs. See Ref. [7] for ODEs of mixed storages and stratified storages.

Georgiev [8] connected a distributed (PDE) collector model and a mixed (ODE) storage model to describe a collector-storage system. After connecting the models of the working components, solar heating systems are generally modelled with ODEs [9–11].

In Refs. [4,12], collector-heat exchanger-storage systems are modelled with a linear (multidimensional) ODE, which will be used in the present paper. This model is validated [4,12,13] or partly validated [14] and accurate enough for different successful applications [10,12,13,15] as well as its improved version in Ref. [13], where the pipes of the system are also modelled with ODEs. The advantage of the basic linear model of Ref. [4] is that it is simpler and easier to use than its extended linear version [13], its nonlinear version [16] or the delay differential equation model of Ref. [15]. The extended linear model of Ref. [13] and the model of Ref. [15] are roughly the same precise and they are more precise than the basic model of Ref. [4]. On the other hand, the extended linear model [13] is simpler and easier to use than its nonlinear version [13] or the model of [15]. Furthermore, the mentioned nonlinear models are approximately the same precise as their linear versions (see Ref. [13], cf [12,16] or see Ref. [17]). Thus the models of [13,15] are the most advantageous in view of accuracy while the linear model of [4] is the most advantageous in view of simplicity. In addition, the latter model is the basis for all other models in Refs. [13,15,16].

From the model of Buzás et al. [9], collector transfer functions have been determined and applied for the dynamic analysis of real collectors [18,19]. The present work extends these results by determining transfer functions for whole solar heating systems and applying them for the dynamic analysis of a real system.

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Nomenclature

A_c	collector surface area (m^2)
A_h	surface area of the heat transfer inside the heat exchanger (m^2)
A_{he}	surface area of the heat exchanger to the environment (m^2)
c_c	specific heat capacity of the collector fluid (J/(kg K))
c_i	specific heat capacity of the fluid in the inlet loop (J/(kg K))
c_h	specific heat capacity of the heat exchanger material (J/(kg K))
I_c	(global) solar irradiance on the plane of the collector (W/m^2)
k_{he}	heat loss coefficient of the heat exchanger to the environment (W/(m^2 K))
m_h	mass of the empty heat exchanger (kg)
t	time (s)
T_c	collector temperature (°C)
T_{ce}	collector environment temperature (°C)
T_{hc}	cold side temperature in the heat exchanger (°C)
T_{he}	environment temperature of the heat exchanger (°C)
T_{hh}	hot side temperature in the heat exchanger (°C)
T_i	inlet (fluid) temperature of the system (°C)
U_L	overall heat loss coefficient of the collector (W/(m^2 K))
v_c	(pump) flow rate in the collector loop (m^3/s)
v_i	(pump) flow rate in the inlet loop (m^3/s)
V_c	collector volume (m^3)
V_h	volume of the heat exchanger (m^3)
εk_h	heat transfer coefficient inside the heat exchanger (W/(m^2 K))
η_0	collector optical efficiency (-)
Φ	heat exchanger effectiveness (-)
ρ_c	mass density of the collector fluid (kg/m^3)
ρ_i	mass density of the fluid in the inlet loop (kg/m^3)

Generally speaking, transfer function based modelling is relatively new and rare in the analysis of solar heating systems, especially, in domestic case. Besides the latter two references, some examples are the following: Amer et al. [20] solved a collector model with time and one space dimension for the fluid temperature using Laplace transformation. Huang and Wang [21] wrote a nonlinear two-node collector model into Laplace transformed form to gain transfer functions. Bettayeb et al. [22] used a two-node model to determine collector transfer functions for the fluid temperature and the absorbed solar energy.

The most prevalent and simple control strategy is the on/off control for solar heating systems in domestic hot water (DHW) production working with constant flow rate, see e.g. Refs. [7,23].

Several controls using pump flow modulation are used in solar heating systems: Winn [24] compared on/off, I (integral) and PID (proportional integral differential) controls, Hirsch [25] compared on/off, P (proportional) and hybrid controls. Löf [26] discussed on/off, P, I, PID, adaptive and certain optimal controls. Optimal controls often maximize the overall energy gain by flow rate modulation. See Refs. [27–29] for the case of no heat exchanger and [30] for the case of a counter flow heat exchanger.

P and PI (proportional integral) controls for collectors are given and a PI control is worked out in details for a real collector field in Ref. [19] based on proposed collector transfer functions. Based on studies in the literature, not many improvements on control for solar heating systems used in domestic applications have been established in the recent few decades. In particular, transfer function based control is rather rare. Pasamontes et al. [31] serve with a further example on the control of the collector field of a solar cooling system based on the transfer function for a mathematical model with time delay.

Transfer function based control is more prevalent for industrial processes e.g. for solar power or desalination plants [32–35].

In the present study, new, validated transfer functions for solar heating systems used primarily for domestic purposes are proposed and used for dynamic analysis and control design. According to a there appointed future research, the present study extends the results of [19], where transfer functions, dynamic analysis and controls have been proposed for solar collectors. The below worked out transfer functions are unique concerning the linear ODE model for solar heating systems in Ref. [4]. If the method for working out transfer functions for this basic model is presented, corresponding transfer functions can be derived rather straightforwardly for the more precise extended linear model of [13] based on the same method. That is why the linear model of [4] is essential and applied in this study to work out transfer functions. (Because of limits in volume, the transfer functions for the extended model of [13] cannot be derived and detailed here.) More precisely, the truncated version of the model of [4] (without modelling the solar storage) is used in the present study, the validation of which is also presented below based on measured data. This means that the here proposed transfer functions are also validated, since they form an alternative representation of the same mathematical model.

The advantages are considerable: Knowing the transfer functions, dynamic analysis of solar heating systems can be carried out and feedback control can be effectively designed based on the well-tried methods of control engineering. A control determined in such a way is generally much simpler than nonlinear and optimal controls and can follow the reference signal much more precisely than the most frequently used on/off controls. Perhaps, the simple applicability is the main advantage of the linear approach concerning transfer functions.

The contributions of the present work in details are the following:

1. Based on a validated and successfully applied ODE model for solar heating systems [4], new transfer functions are mathematically derived (at the end of Section 3.1) and validated. The applicability of the transfer functions are interpreted with the dynamic analysis of a real system.
2. As a further important application of the transfer functions, closed-loop (PI) control design is given with stability criteria for solar heating systems using the methods of control engineering. The efficiency of the proposed control design is demonstrated based on simulation results.

The present paper proposes all the concepts of work [19] (transfer functions, dynamic analysis, control design) for whole solar heating systems and not for collectors alone, so the present contributions are even more important and more general than the results of [19].

The paper is organized as follows: Section 2 describes the model for solar heating systems, for which the transfer functions are