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Geometric properties of the single-diode photovoltaic model and a new very simple method for parameters extraction

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ABSTRACT

One of the most important models to predict the electrical behavior of a photovoltaic (PV) module is the so-called single-diode model. This model is derived from the electrical equivalent circuit formed by a current source in parallel with one diode, a shunt resistor and a series resistor. The model equation depends on five parameters, if these parameters are obtained, it has been tested that this model fits accurately the real behavior of the PV module under a minimum of illumination. Nevertheless, the extraction of the parameters is quite difficult since none of the variables in the model equation can be expressed in explicit form. This fact also implies the difficulty of knowing the real properties of the current as an implicit function of the voltage in this model. Knowing these properties deeply will involve a more suitable use of the model and better understanding of the behavior of the photovoltaic module. The first goal of this paper is the rigorous mathematical study of the model. In particular, it is demonstrated that the model equation actually defines the current as an implicit function of the voltage which is indefinitely differentiable along the real line. We will provide the most significant geometric properties of the current function by means of the study of the first and the second derivatives functions which are also implicitly given. The second goal of the paper is, given real data of voltage and current measured from a PV module, to show how the parameters of the model equation can be extracted in a very simple way, giving rise to an estimated curve which fits accurately the real one. The proposed new analytical method is as good, for instance, as the well-known analytical five point method but significantly simpler.

1. Introduction

The single-diode equivalent electrical circuit of a photovoltaic (PV) module with N_p cells in parallel and N_s cells in series is depicted in Fig. 1.

At a given illumination, the relationship between the current i (measured in Amperes [A]) and the voltage v (measured in Volts [V]) obtained from the previous circuit is given by the model equation

$$i = N_p I_{ph} - N_p I_{sat}\left(\exp\left(\frac{1}{nV_T}\left(\frac{v}{N_s}+\frac{iR_s}{N_p}\right)\right)-1\right) - N_p\frac{1}{R_{sh}}\left(\frac{v}{N_s}+\frac{iR_s}{N_p}\right) \quad (1)$$

where I_{ph} is the photocurrent measured in A; I_{sat} is the diode saturation current measured in A; R_s is the series resistance measured in Ohms [Ω]; R_{sh} is the shunt resistance measured in Ω; n is the diode ideality factor; $V_T = (k/q)T$, where T is the temperature of the cells measured in Kelvin degrees [K], $k = 1.3806488 \times 10^{-23}$ is the Boltzmann's constant measured in Joules per Kelvin [J]/[K] and, $q = 1.60217653 \times 10^{-19}$ is the electronic charge measured in Coulombs [C].

The solutions (v, i) of the previous equation generate a curve in the Euclidean plane. We will refer to this theoretical curve as i–v curve to distinguish it from the real I–V curve (or I–V characteristic) generated by the PV module.

The theoretical i–v curve fits the real I–V characteristic of most of the PV modules with very good accuracy (see, for instance, Refs. [1,2]) under a minimum of illumination (about half AM1 according to [2]). If the five parameters I_{ph}, I_{sat}, n, R_s, and R_{sh} are extracted for certain conditions of irradiance and temperature, the electrical behavior of the PV module can be precisely predicted for any other conditions. A large amount of papers (see, among others, Refs. [3–28]) have dealt with the problem of

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extracting the previous parameters using different methodologies. Such a quantity of papers dealing with this model demonstrate its importance and its acceptance for modeling a PV module.

Some of the previous reported methodologies consist on solving a system of five non-linear equations obtained after substituting real data, measured on the I–V curve, on the model equation and on the equation obtained derivating the previous one (see, Refs. [4–6,18]). Another kind of methodology consists on obtaining the best theoretical i–v curve fitting the real one by solving an optimization problem [2,7–10,13,24]. There are also methods which use non-elementary functions as the Lambert W function to extract the parameters in a different way [19,23,25].

Some papers dealing with simplified models of three and four parameters [26–28], have been able to obtain exactly and explicitly the corresponding parameters just by using information of the I–V curve near the operating point of the PV module.

We would like to point out that to properly use a model, it is important to know its mathematical properties such as differentiability, boundedness, sign of the function and its derivatives, asymptotic behavior, etc. which provide its geometric properties. The knowledge of these properties help researchers to better understand the electrical behavior of the PV modules, as well as, to study new techniques to extract the parameters. This paper aims to study deeply some of these properties and, taking the advantage of the theoretical knowledge, to provide a new method to obtain easily, quickly and accurately the parameters of the single-diode model taking real measurements of a PV module.

Before starting with the mentioned results, let us rewrite Eq. (1) in a more simplified way which depends on new five parameters gathering the original ones in a reduced form.

If we denote

$$\begin{aligned} A &:= \frac{N_p I_{ph} R_{sh}}{R_{sh}+R_s} \\ B &:= \frac{N_p I_s R_{sh}}{R_{sh}+R_s} \\ C &:= \exp\left(\frac{1}{N_s n V_T}\right) \\ D &:= \exp\left(\frac{R_s}{N_p n V_T}\right) \\ E &:= \frac{N_p}{N_s}\,\frac{1}{R_{sh}+R_s} \end{aligned} \tag{2}$$

Eq. (1) becomes

$$i = A - B\left(C^v D^i - 1\right) - Ev \tag{3}$$

Since the parameters I_{ph}, I_{sat}, n, R_s, and R_{sh} are strictly positive, the constants in the previous equation satisfy

$$A>0, B>0, C>1, D>1, E>0 \tag{4}$$

If the new parameters are extracted, the original ones can be retrieved as:

$$\begin{aligned} I_{ph} &= \frac{1}{N_p}\,\frac{A\ln C}{\ln C - E\ln D} \\ I_s &= \frac{1}{N_p}\,\frac{B\ln C}{\ln C - E\ln D} \\ nV_T &= \frac{1}{N_s}\,\frac{1}{\ln C} \\ R_s &= \frac{N_p}{N_s}\,\frac{\ln D}{\ln C} \\ R_{sh} &= \frac{N_p}{N_s}\left(\frac{1}{E} - \frac{\ln D}{\ln C}\right) \end{aligned} \tag{5}$$

2. Properties of the intensity function defined by the single-diode model

In this section, let us prove that for a voltage v, the model equation (3) gives a unique intensity value i providing then a real function $i(v)$ which can be derivated as many times as wanted at any voltage v. We will prove that $i(v)$ is strictly decreasing and strictly concave. Moreover, we will see that the second derivative function has a local (indeed global) minimum. The behavior of the functions i, i' and i'' when v tends to ∞ will also be studied showing that function i' has horizontal asymptotes at right and left and, function i has an oblique asymptote at left which is often perceptible near the short circuit point. This asymptotic behavior will give the key tool to develop a new analytical method to extract the parameters of the single-diode model.

2.1. Single-diode model equation defines i as a function of v

First, let us prove that variable i in Eq. (3) can be seen as a function of v along the real line R which is, moreover, indefinitely differentiable at any point of R.

Given an open subset Λ of the n-dimensional Euclidean space R^n, a real function defined in Λ is said to be of class $C^\infty(\Lambda)$ if it has continuous partial derivatives of any order at any point of Λ.

Theorem 1. *Eq. (3) defines i as an implicit function of v of class $C^\infty(R)$.*

Proof. Consider the function $F(v,i) := i - A + B(C^v D^i - 1) + Ev$. Observe that $F(v,i) = 0$ if and only if (v,i) is a solution of (3).

Let us divide the proof into two steps.

Step 1: Eq. (3) defines i as a function of v.

Given $v_0 \in R$, define the function $F_{v_0}(i) := F(v_0, i)$ which is differentiable with $F'_{v_0}(i) = 1 + BC^{v_0}D^i \ln D + E > 0$.

For $v_0 \geq 0$, taking $i_1 := -Ev_0 - v_0 \log_D C$ and $i_2 := A + B$ one has $F_{v_0}(i_1) < 0$ *and* $F_{v_0}(i_2) > 0$.

For $v_0 < 0$, taking $i_3 := v_0 \log_D C$ and $i_4 := A + B - Ev_0$ one has $F_{v_0}(i_3) < 0$ and $F_{v_0}(i_4) > 0$.

From Bolzano's theorem and the fact that F_{v_0} is strictly increasing, one has for any $v_0 \in R$ a unique $i_0 \in R$ such that $F(v_0, i_0) = F_{v_0}(i_0) = 0$. So, we can define the function $i : R \to R$ which assigns to each v the unique i satisfying $F(v,i) = 0$.