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# Modeling for control of a kinematic wobble-yoke Stirling engine[☆]

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## ABSTRACT

In this paper we derive the dynamical model of a four-cylinder double-acting wobble-yoke Stirling engine. In addition to the classical thermodynamics methods that dominate the literature of Stirling mechanisms, we present a control systems viewpoint to analyze the dynamic properties of the engine. We show that the Stirling engine can be viewed as a closed-loop system, in which the pressure variations in the cylinders behave as the feedback control law.

Energy savings and concern for the environment and climate are major issues nowadays within our society. Due to the high costs of extraction and processing of fossil fuels—which have made their utilization increasingly expensive [1–3], not to mention the adverse effects to the environment—, sustainable energies such as wind and solar energy are becoming popular around the world [4–8]. Moreover, in recent decades there has been an enormous interest in the application of heat engines for converting different types of heat sources into electrical energy [9,10].

One of the most promising applications is micro-combined heat and power (CHP) generation, or in other words, the simultaneous production of heat and power at a small-scale [11]. A micro-CHP consists of a gas engine which drives an electrical generator. Among the advantages of using micro-CHP systems we can mention: cutting the power transmission losses, because the waste heat can be captured and used locally; and generating electricity that can be either used in the house or exported to the grid in order to be consumed by the neighbors [11]. Supplying electricity back to the grid raises important economical and research/scientific challenges [10] which are not within the scope of this paper (see for instance [12] and references therein).

Micro-CHP systems can attain a similar conversion efficiency from gas to useful heat as a conventional boiler, typically around 80%. However, in addition, around 10–15% can be converted to electricity. Among the technologies that have been proposed for micro-CHP applications we can mention fuel cells, internal combustion engines and Stirling engines [11,13]. The Stirling engine is an external combustion reciprocating engine invented by Robert Stirling in 1816. Theoretically, Stirling engines seem to be the most efficient device for converting heat into mechanical work, with high efficiencies, requiring high-temperatures [14]. Stirling engines are generally externally heated engines. Therefore, most sources of heat can be used to drive them.

Because of the Stirling engine inherent complexity, providing modeling and simulation tools for improving its design has raised important research challenges for the scientific community during the last decades. Studies that rely on thermodynamics methods and intuitive design techniques can be found in Refs. [15–19]. In Refs. [20–23], the application of computational fluid dynamics modeling to improve the design of Stirling engines is discussed. There exist, however, few literature on the application of systems and control methods to investigate their stability and dynamic properties, see for instance [24–28] and the recent works [29,30].

In this work we present a dynamic systems and control perspective to analyze the complex behavior of the Stirling engine. To this end, we take as a case study a kinematic wobble-yoke Stirling engine [31–33], consisting of a four-cylinder double-acting Stirling mechanism whose design is based on the classical spherical four-bar linkage. Our contributions are threefold. First, we present the complete nonlinear dynamical model of the kinematic wobble-yoke Stirling engine (originally developed by the authors in Ref. [34]). Second, we show that the Stirling engine can be viewed as a

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closed-loop feedback system, in which the pressure variation inside the cylinders behaves as the state feedback control law. Third, following a similar approach from Refs. [29,35], we investigate the dynamic and stability properties of the Stirling engine.

The paper is organized as follows. Section 1 describes the working principle of the kinematic wobble-yoke Stirling engine. Section 2 introduces the dynamic modeling of the engine. In Section 3 the linear dynamics of the engine is analyzed. Section 4 presents the application of linear control tools to study the behavior of the Stirling engine. Simulations results are given in Section 5 and finally Section 6 outlines some concluding remarks.

## 1. Description of the system

Fig. 1 shows the schematic representation of the four-cylinder double-acting Stirling engine. The four cylinders are phased at $90^o$ from each other with respect to $\phi$. The links connecting the cylinders form the wobble-yoke mechanism whose function is to translate the reciprocating motion in vertical direction of the cylinders into the rotational motion through the shaft angle $\phi$. The design of the wobble yoke mechanism is based on the classical spherical four-bar linkage [31]. These kind of linkages, which are well known in robotics, have the property that every link in the system rotates about the same fixed point [36,37]. Hence, as indicated by its name, the trajectories of the points at the end of each link lie on concentric spheres. In robotics, only the revolute joint is compatible with this rotational movement and its axis must pass through the fixed point. The wobble yoke is indeed a particular class of the spherical linkage known as *spherical crank rocker* [31]. In this case, the revolute joints are replaced by the spherical bearings located at points $b_1$, $b_3$, $c_1$ and $d$ (cf. Fig. 2). The axis of the aforementioned bearings must intersect the sphere center $O$.

The working principle of this mechanism can be explained by referring to Fig. 2. The mechanism is based on a beam which pivots about its center $O$ in one plane ($e_2e_3$ for beam 1, and $e_1e_3$ for beam 2). Each beam is attached to pistons via bearings $a_1$ and $a_3$. An eccentric bearing $c_1$ is attached to the drive shaft and it is connected to the beam via two bearings $b_1$ and $b_3$. The eccentric bearing $c_1$ is the rotating part of the mechanism. When the engine is working, the reciprocating motion in vertical direction of the pistons inside the cylinders (not shown in Fig. 2), induces a rotational movement on bearing $c_1$. Due to the geometrical and physical configuration of the mechanism, bearing $c_1$ describes a circle of radius $l_{c_1d}$. The axis of bearings $b_1$, $b_3$, $c_1$ and $d$ must intersect the center $O$, so that the kinematic constraints of the spherical crank rocker [36,37] are satisfied. We also notice that the axis $l_{Oc_1}$ of bearing $c_1$ is perpendicular to the beam, i.e., $l_{Oc_1} \perp l_{b_1b_3}$. An analogous discussion applies to the second beam. We refer the reader to [31,32] for more details about the wobble-yoke Stirling engine.

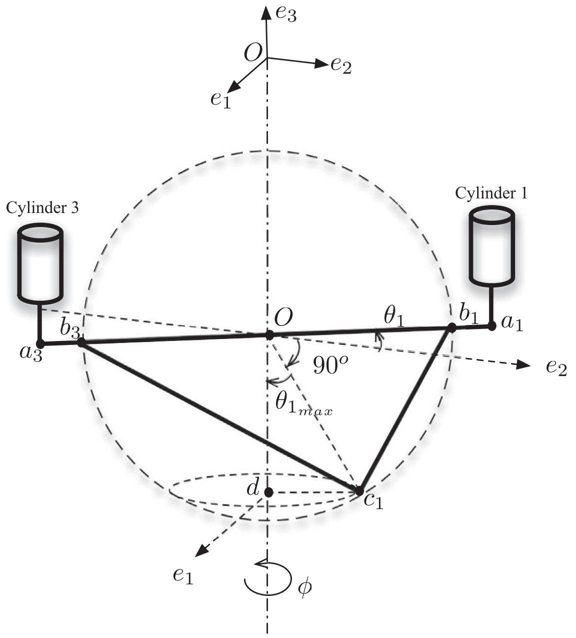

**Fig. 2.** Schematic picture of beam 1. $\theta_1$ is the angle between the beam and the axis $e_2$. The angle $\phi$ is measured in the counterclockwise direction from the positive axis $e_1$.

## 2. Modeling for control

In this section we derive the equations of motion for the wobble-yoke Stirling engine [34]. The definition of the parameters as well as their nominal values are summarized in Table 1. We make the following fundamental assumption:

A1. *Small motion*: Let $-15^\circ < \theta_j < 15^\circ$, then $\cos\theta_j \approx 1$, $\sin\theta_j \approx \theta_j$, $\dot{\theta}_j^2 \approx 0$.

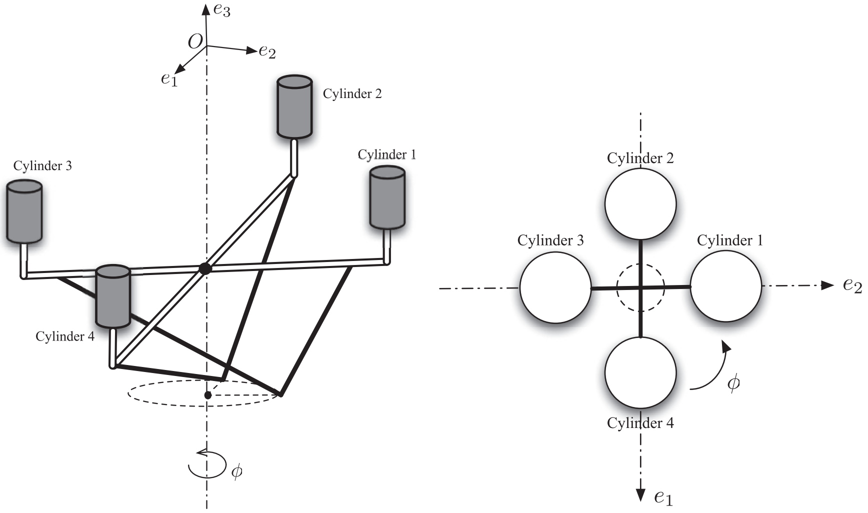

**Fig. 1.** Schematic representation and cylinders configuration of the wobble-yoke Stirling engine.