Renewable Energy 69 (2014) 174–179

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Technical note

Vortical structures in the wake of the savonius wind turbine by the discrete vortex method

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ARTICLE INFO

Article history:
Received 24 October 2013
Accepted 19 March 2014
Available online

Keywords:
Savonius
Wind turbine
Numerical modelling
Vortex shedding
Discrete vortex method

ABSTRACT

This paper treats the vortex shedding phenomenon of a savonius wind turbine, whose knowledge is primordial in correctly calculating the airloads on the blades. The specific aim being to numerically predict the disposition and geometry of the vortical structures in the wake of the savonius rotor whose existence has been visualised by a number of experimentalists. In the numerical approach, the blade is represented by discrete bound vortices while the wake is generated in a time stepping calculation as an emission of free vortices. The calculations are enhanced by the Newmann boundary condition coupled to the Kutta–Joukowsky condition and the Kelvin's theorem for the conservation of circulation. The convection of the vortices in the wake is accomplished through a predictor corrector integration scheme. A code has been developed which predicts the wake structure to be in good agreement with the experimental visualizations: For low tip speed ratios, the wake consists of a series of three discrete vortical structures while at higher tip speed ratios, the characteristic structure is the presences of a central vortex.

1. Introduction

The savonius rotor is a vertical axis wind turbine built up of two half cylinders displaced along their common diameter. It is very robust, simple to construct and is characterised by a high starting torque. Fig. 1a and b, respectively, shows a perspective view and a view in the median plane of a savonius rotor.

Past visualisation studies conducted on the savonius rotor lead to two main conclusions concerning the vortical structures in its wake. In the first case, the wake consists of the formation of three distinct vortical structures for each complete turn of the rotor (Benghrib et al. [2], Botrini et al. [3]). In the second case the wake is characterised by the presence of a central vortex located at the centre of the rotor (Modi et al. [14]). These two scenarios will be investigated numerically in the present paper.

The flow field around the savonius rotor is unsteady and nonlinear owing to the interaction of the air with the rotating rotor. The unsteady inviscid potential theory will be applied in the modelling. The calculations will be carried out on a 2D cut of the rotor in the median plane (Fig. 1b). The rotor is represented in the median plane by two half circles displaced along their common diameter by a small distance denoted by e. Each half circle is then treated as a lifting airfoil in interaction with the other (Fig. 2).

The modelling is carried out using the discrete vortex method whereby the contour of the two half circles is represented by a distribution of bound vortices. The unknown intensities of these vortices are obtained by applying the Neumann boundary condition together with the Kutta–Joukowsky condition. The Kelvin theorem assures the conservation of the circulation in the flow field.

The unsteady motion of an inviscid fluid about an airfoil is always accompanied by the shedding of the boundary layer, which extends in to the fluid from the separation point in the form of a thin sheet. As the separated boundary layer moves along, it is deformed in such a way that vortices are generated along it. These vortices gradually diffuse and disperse into the flow field. This phenomenon is actually observed in the case of small viscosity (high Reynolds number). It is possible to obtain a reasonable close mathematical description for the study of an ideal fluid motion by using the potential theory Afungchui et al. [1], Kamoun et al. [10], Leishman [11].

A theoretical study has earlier been carried out on the savonius rotor by Ogawa [15], but this work was rather general and special

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http://dx.doi.org/10.1016/j.renene.2014.03.038
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Nomenclature

$\times$	vector product
$\overrightarrow{U_0}$	upcoming wind velocity
S	incepted surface of the rotor
l	length of a segment of the baseline profile
$\overrightarrow{r}$	position vector of a point on the profile
R	rotor diameter
$\overrightarrow{n}$	normal unit vector
$\overrightarrow{s}$	tangential unit vector
t	time instant
Δt	time increment
r_c	vortex core radius
z	coordinates of a point in the complex plane
f	complex potential
d	diameter of the semicircle
D	diameter of the savonius rotor
e	displaced distance of two half circles
H	height of rotor

Greek symbols

γ	bound vortex intensity
θ	angular position
η	parameter which can take A or B
Γ	free vortex intensity
λ_0	tip speed ratio ($\lambda_0 = \omega D/(2U_0)$)
$\overrightarrow{\omega} = \overrightarrow{\alpha}$	angular velocity
Ψ	stream function
Φ	potential function

Subscript

A	semicircle A
B	semicircle B
LE	leading edge
TE	trailing edge
ALE	leading edge of A
ATE	trailing edge of A
BLE	leading edge of B
BTE	trailing edge of B
AE	one of the extremities of A
BE	one of the extremities of B
WAE	wake point emitted from one of the extremities of A
WBE	wake point emitted from one of the extremities of B
i	receptor point
j	inductor point

emphasis was not laid on the vortex formation as predicted by experimental visualizations. This paper will treat in detail this aspect of the flow over the rotor. It is well known from numerical simulations that two-dimensional dynamics of flow over bodies are distinguished by the presence of long-lived organized coherent shears and vortices which are characterized by high energy and vorticity concentrations. The contribution on the dynamics of coherent structures, including their time-evolution and interactions, plays an important role to the understanding of the spatial flows structure. Coherent vortices develop basic inhomogeneities and nonlocal dynamical properties which, in general, cannot be considered in phenomenological analyses or within the framework of statistical approaches. Consequently, the Objective of this paper is to numerically predict the geometry of the vortical structures in the wake of the savonius rotor which has a direct effect on its aerodynamic performance. The formulation of the problem is based on the discrete vortex method as outlined in what follows.

2. Methods

The contour of each half circle is divided into a certain number of segments. Discrete vortices are distributed at the middle of the segments while the control points are located at the extremities of the segments.

The leading edge (LE) and the trailing edge (TE) of each half circle are the points of the emission of the free vortices. The vortex intensities on the half circles are continuously changing in time and are determined by applying the flow tangency condition; Euvrand [5], Giesing [7,8], together with the Kutta–Joukowsky (KJ) condition; Afungchui et al. [1], Euvrand [5], Herman [9], Maskell [12], Poling et al. [16]. The points where the vortices are emitted are fixed in advance at some little distances from the extremities of the half circles. All these aspects are represented in Fig. 2. The free vortex intensities emitted at each time instant from the four extremities of the half circles, at the nascent points, are obtained by applying the circulation conservation theorem of Kelvin Max [13], Poling and Telionis [16], Sipcic and Morino [17], Suciu and Morino [18]. This condition provides a means for the shedding of the free vortices into the wake.

If each half circle is divided into N line segments, then there will be $(2N + 4)$ equations to solve at each time step in order to determine the $(2N + 4)$ unknowns. These unknowns are the intensities of the N bound vortices on each of the two half circles and the four free vortices emitted at the nascent points.

The action of an up coming wind with a velocity $\overrightarrow{U}_0$, brings about the rotation of the rotor about its centre with an angular velocity $\overrightarrow{\omega}$. The absolute velocity is irrotational and can be derived from a potential function Φ. The complex potential $f(z)$ of the flow field, at a point z on one of the half circles, at any time instant is given by Ref. [1]:

$$
\begin{aligned}
f(z) = {} & \sum_{j=1}^{N} \frac{I(\gamma_A)_j}{2\pi} Log\left[z - (z_A)_j\right] + \sum_{k=1}^{2} \frac{I(\Gamma_{AE})_k}{2\pi} Log\left[z - (z_{AE})_k\right] \\
& + \sum_{k=1}^{2} \sum_{j=1}^{n-1} \frac{I(\Gamma_{wAE})_{kj}}{2\pi} Log\left[z - (z_{wAE})_{kj}\right] \\
& + \sum_{j=1}^{N} \frac{I(\gamma_B)_j}{2\pi} Log\left[z - (z_B)_j\right] + \sum_{k=1}^{2} \frac{I(\Gamma_{BE})_k}{2\pi} Log\left[z - (z_{BE})_k\right] \\
& + \sum_{k=1}^{2} \sum_{j=1}^{n-1} \frac{I(\Gamma_{wBE})_{kj}}{2\pi} Log\left[z - (z_{wBE})_{kj}\right] \\
& + U_0 z - \omega xy - I\frac{\omega}{2}\left(x^2 + y^2\right)
\end{aligned} \tag{1}
$$

where $z=(x + Iy)$ is the coordinate of a point in the complex plane, $I = \sqrt{-1}$, the summation index (k), represents the two extremities of a semicircle, n is the number of time steps since the beginning of motion, N is the number of segments of each semicircle and U_0 is the upstream velocity considered to be a complex quantity. The potential of the flow is simply deduced as being the real part of $f(z)$.

The main boundary condition is the flow tangency condition which enforces the semicircle to agree with one of the relative streamlines. This condition is implemented by annulling the