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A dimensional analysis for determining optimal discharge and penstock diameter in impulse and reaction water turbines

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ABSTRACT

This paper presents a dimensional analysis for determining optimal flow discharge and optimal penstock diameter when designing impulse and reaction turbines for hydropower systems. The aim of this analysis is to provide general insights for minimizing water consumption when producing hydropower. This analysis is based on the geometric and hydraulic characteristics of the penstock, the total hydraulic head and the desired power production. As part of this analysis, various dimensionless relationships between power production, flow discharge and head losses were derived. These relationships were used to withdraw general insights on determining optimal flow discharge and optimal penstock diameter. For instance, it was found that for minimizing water consumption, the ratio of head loss to gross head should not exceed about 15%. Two examples of application are presented to illustrate the procedure for determining optimal flow discharge and optimal penstock diameter for impulse and reaction turbines.

1. Introduction

The world energy consumption will grow by 56% between 2010 and 2040 [6]. As world population continues to grow and the limited amount of fossil fuels begins to diminish, there is an increasing demand to exploit renewable sources of energy.

In the United States, about 9% of all energy consumed in 2012 was from renewable sources [7]. While this is a relatively small fraction of the U.S. energy supply in 2012, the United States was the world's largest consumer of renewable energy from geothermal, solar, wood, wind, and waste for electric power generation producing almost 25% of the world's total [7]. This institute also reports that in 2012, 30% of the renewable energy in the U.S. was from hydropower. This means that only about 3% of all energy consumed in the United States was from hydropower.

Globally, hydropower accounted for 16% of all global electricity production in 2007, with other renewable energy sources totaling 3% [5]. Hence, it is not surprising that when options are evaluated for new energy developments, there is strong impulse for fossil fuel or nuclear energy as opposed to renewable sources. However, as hydropower schemes are often part of a multipurpose water resources development project, they can often help to finance other important functions of the project [3]. In addition, hydropower provides benefits that are rarely found in other sources of energy. In fact, dams built for hydropower schemes, and their associated reservoirs, provide human well-being benefits, such as securing water supply, flood control and irrigation for food production, and societal benefits such as increased recreational activities and improved navigation [3].

Furthermore, hydropower due to its associated reservoir storage, can provide flexibility and reliability for energy production in integrated energy systems. The storage capability of hydropower systems can be seen as a regulating mechanism by which other diffuse and variable renewable energy sources (wind, wave, solar) can play a larger role in providing electric power of commercial quality [5]. While development of all the remaining hydroelectric potential could not hope to cover total future world demand for electricity, implementation of the remaining potential can make a vast contribution to improving living standards in the developing world (South America, Asia and Africa), where the greatest potential still exists [7].

Minimizing water consumption for producing hydropower is critical given that overuse of flows for energy production may result in a shortage of flows for other purposes such as irrigation or navigation. The present work was motivated when the first author was unable to find in the literature a theoretical framework for determining optimal flow discharge and optimal penstock diameter for the design of impulse and reaction turbines. Recently, Pelz

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[4] provided a theoretical approach for determining the upper limit for hydropower gained by a water wheel or turbine per unit width in a rectangular open-channel. This is somewhat different of impulse and reaction turbines, as in the latter turbines, the flow in the penstock is pressurized.

This paper aims to provide general insights on determining optimal flows and optimal penstock diameters when designing impulse and reaction turbines for hydropower systems. This paper is divided as follows. First, dimensionless relationships between power production, flow discharge and head losses are derived. Second, these relationships are used to withdraw general insights on determining optimal flow discharge and optimal penstock diameter. Third, examples of application for determining optimal flows when designing impulse and reaction turbines are presented. Finally, the key results are summarized in the conclusion.

2. Dimensional analysis for optimal flow discharge, optimal head losses and optimal power

The electric power, P, in Watts (W), can be determined by the following equation:

$$P = \eta\gamma Q(H_g - h_L) \tag{1}$$

where γ $(= \rho \times g)$ is specific weight of water in kg/(m^2 $\times$ s^2), Q is flow discharge in m^3/s, H_g is gross head in m, h_L is sum of head losses in m, ρ is water density in kg/m^3, g is acceleration of gravity in m/s^2, and η is overall hydroelectric unit efficiency, which in turn is the product of turbine efficiency (η_t) and generator efficiency(η_g). In all derivations presented in this paper, it is assumed that η $(= \eta_t \times \eta_g)$ is constant.

For an impulse turbine (see Fig. 1), the sum of head losses can be written as

$$h_L = \frac{Q^2}{2gA_2^2}\left[f\frac{L}{D_2} + \sum k_{1-2} + k_N\left(\frac{A_2}{A_N}\right)^2\right] \tag{2}$$

where L, D_2 and A_2 are length, diameter and cross-sectional area of penstock, respectively. In addition, f is friction factor, $\sum k_{1-2}$ is the sum of local losses in penstock due to entrance, bends, penstock fittings and gates, A_N is nozzle area at its exit (section 3 in Fig. 1) and k_N is nozzle head loss coefficient, which is given by (e.g., [1]).

$$k_N = \frac{1}{C_V^2} - 1 \tag{3}$$

where C_V is nozzle velocity coefficient. According to Dixon [2], C_V varies between 0.98 and 0.99 for a typical Pelton turbine nozzle.

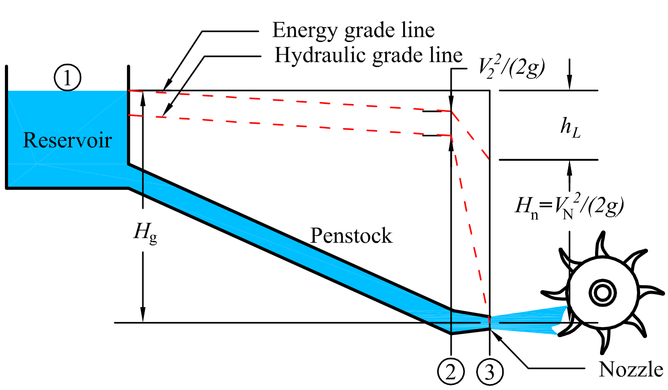

Fig. 1. Sketch of an impulse turbine.

For a reaction turbine (see Fig. 2), the sum of head losses can be written as

$$h_L = \frac{Q^2}{2gA_2^2}\left[f\frac{L}{D_2} + \sum k_{1-2} + \left(\frac{A_2}{A_d}\right)^2\right] \tag{4}$$

where A_d is draft tube cross-sectional area at its outlet (Section 3 in Fig. 2).

The expression inside the brackets in Eqs. (2) and (4) is dimensionless and it is denoted herein as

$$C_L = \begin{cases} f\dfrac{L}{D_2} + \sum k_{1-2} + k_N\left(\dfrac{A_2}{A_N}\right)^2 & \text{for an impulse turbine} \\ f\dfrac{L}{D_2} + \sum k_{1-2} + \left(\dfrac{A_2}{A_d}\right)^2 & \text{for a reaction turbine} \end{cases} \tag{5}$$

Hence, the total head losses in Eqs. (2) and (4) is equal to the product of C_L and $Q^2/(2gA_2^2)$ and thus, Eq. (1) can be written as

$$P = \eta\gamma Q\left(H_g - C_L\frac{Q^2}{2gA_2^2}\right) \tag{6}$$

For generalizing the findings in this paper, a dimensionless relationship between power and flow discharge is sought. To achieve this, Eq. (6) is divided by a reference power (P_r). P_r is assumed to be the maximum power that can be generated using a reference discharge (Q_r) and a fixed gross head and penstock geometry (constant C_L). For maximum power, the turbine and generator efficiencies need to be 100% (i.e., η_t = 100% and η_g = 100%). Also, maximum power for a fixed penstock geometry can be obtained by setting dP/dQ in Eq. (6) equal to zero, which gives

$$h_L = \frac{H_g}{3} \tag{7}$$

The reference flow discharge Q_r can be obtained by using Eq. (7) and the energy equation between the reservoir and the nozzle exit for an impulse turbine or between the reservoir and the tailrace for a reaction turbine, which gives:

$$Q_r = 2A_3\sqrt{\frac{1}{3}gH_g} \tag{8}$$

where A_3 is the cross-sectional area at Section 3 in Figs. 1 and 2, given by

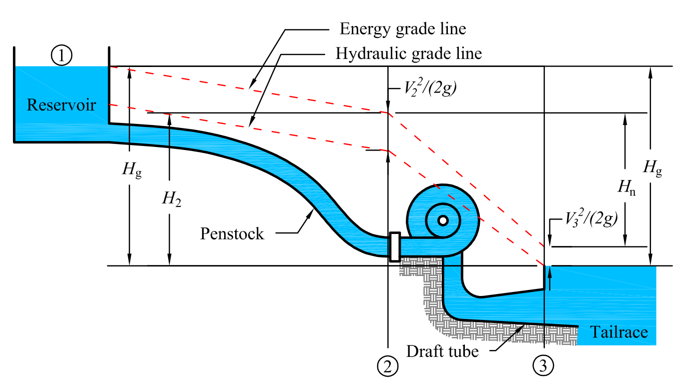

Fig. 2. Sketch of a reaction turbine.