Renewable Energy 63 (2014) 797–806

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Power extraction from tidal channels – Multiple tidal constituents, compound tides and overtides

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ARTICLE INFO

Article history:
Received 31 May 2013
Accepted 19 October 2013
Available online

Keywords:
Tidal stream energy
Resource assessment
Overtide
Spring/neap cycle
Pentland Firth

ABSTRACT

Many candidate sites for tidal stream power extraction are tidal channels, and the power that can be generated from these sites will be directly related to the amplitude and phase of the principal tidal constituents driving flow through the channel. This paper investigates this interaction between energy extraction and tidal constituents, and also the effect that power extraction may have on harmonics of the principal constituents (i.e. compound tides and overtides). Firstly, the variation in power extraction and available power (defined as the fraction of extracted power removed by idealised tidal turbines) are investigated over a spring/neap tidal cycle using a simple theoretical model. Results from the model are used to derive analytical bounds to the variation in power at spring and neap tide. These bounds are shown to depend on the channels natural dynamic balance and are of practical importance to tidal stream device developers looking to supply power to the electricity grid. Secondly, changes in the higher harmonics in channel flow rate are investigated for deployments of tidal farm in channels of various length and geometry. Specifically, it is shown that in general if the turbines provide a uniform drag resistance to the flow through the channel, even harmonics in the flow rate will decay with power extraction (leading to a more symmetric tide), whilst odd harmonics in the flow rate may decay or increase depending on the natural tidal dynamics. These variations can have significant effect on residual flows and the local environment. Throughout the paper results from the theoretical model are compared with a complex numerical model of energy extraction from the Pentland Firth. Good agreement is shown in all cases.

1. Introduction

An ideal location to install tidal stream energy devices is in a tidal channel with fast flowing tidal currents. A well-known example of this type of site is the Pentland Firth in the UK. Recent estimates for this location have concluded that theoretically an average power extraction of up to approximately 4 GW could be removed [1], or between 5 and 10% of the UK's current annual electricity generation. This estimate is, however, only an upper bound and does not account for many practical constraints to generating tidal power. Recently Adcock et al. [2] have revised the upper bound estimate for the Pentland Firth down to ~1.9 GW by modelling idealised tidal turbines in the Pentland Firth using actuator disc theory. This approach allowed for the hydrodynamic efficiency of turbines to be included so as to estimate the fraction of extracted power that is available to the turbine (with the fraction defined as available power, following [3]). However [2], also points out that, due to the presence of multiple tidal constituents, the peak daily average power generated by tidal devices will vary significantly over the entire spring/neap tidal cycle. This variation has implications for the design of tidal turbines and can have implications on the usefulness of generated power for supply to the UK electricity grid.

In addition to the variations in power generation due to multiple tidal constituents, it is well known that harmonics of the principal constituents, such as compound tides and overtides, are also common in shallow water locations with significant tidal movement. These non-linear tidal components are a result of interactions between the principal constituents and play an important role in the net transport of pollutants and seabed sediment [4]. It is therefore of practical importance to understand how power extraction may interact with tidal harmonics.

Motivated by these observations, the purpose of this paper is to explore the importance of multiple tidal constituents, compound tides and overtides, on tidal stream power extraction (and *vice versa*) in a tidal channel. To keep the analysis simple we consider

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http://dx.doi.org/10.1016/j.renene.2013.10.037

the extraction of power from an idealised tidal channel in which the elevation difference driving flow through the channel is unaffected by any changes to the flow through the channel. This model follows from the theoretical model of Garrett & Cummins [5] and is a simplification of the complex dynamics of real tidal sites such as the Pentland Firth, but gives valuable insight into how energy can be extracted from tidal channels [6]).

Using the model we examine channels driven by multiple tidal components, including how tidal power extraction will change over the spring/neap tidal cycle, and how available power will change assuming idealised (inviscid) turbines modelled using actuator disc theory. We also use the model to investigate the effect of tidal power extraction on the magnitude of tidal harmonics.

We support our analysis by comparing results from the model with a depth-averaged numerical model of energy extraction from the Pentland Firth.

2. Models

2.1. Channel model

In this paper we adopt a simple model for the tidal dynamics in a channel, which is based on that given in Garrett & Cummins [5]. Whilst the model does not account for the complex nature of the flow in a real channel, it gives a conceptual model which gives a useful insight into how energy may be extracted from tidal streams at actual coastal sites [6,7]. It gives rather poorer agreement with estimates of available power from complex actual sites [2]. This is due to the variation in flows across the width of the channel not accounted for in the simple model (See Section 5). Nevertheless, the model can be used to draw general conclusions as to how the available power will vary at real sites.

To develop the model we consider a narrow tidal channel (see Fig. 1), which has a cross-sectional area $A(x)$ which varies along its length. The 1D shallow water continuity equation and approximation to the momentum equation are used to describe flow through the channel. These are given by:

$$\frac{\partial\zeta}{\partial t}+\frac{1}{b}\frac{\partial(Q)}{\partial x} = 0, \tag{1}$$

and

$$\frac{\partial}{\partial t}\left(\frac{Q}{A}\right)+\left(\frac{Q}{A}\right)\frac{\partial}{\partial x}\left(\frac{Q}{A}\right)+g\frac{\partial\zeta}{\partial x} = -F_{\text{nat}}-F_{\text{turbine}}, \tag{2}$$

where ζ is the free surface elevation above still water level, b is the breadth of the channel, and g is acceleration due to gravity. The terms on the right hand side of equation (2) represent forces (per unit mass) and include a seabed drag component, F_{nat}, which represents the naturally occurring friction (taken herein to be $F_{\text{nat}} = \frac{1}{2}C_d Q|Q|/A^2 h$, where h is the water depth and C_d is a bed friction parameter) and F_{turbine}, which is the total thrust applied by turbines to the flow through the channel.

As pointed out by Refs. [5] and [8], in most actual tidal channels (i.e. those which are short compared with the principal tidal wavelength) the volumetric flow rate, Q, will be approximately constant along the channel (which follows from (1)). In addition, the variation in water level relative to the water depth along the channel is often small so that the cross-sectional area, A, does not vary in time. Integrating (2) along the channel therefore leads to

$$c\frac{dQ}{dt}=g\eta-\left(\delta_0+\sum_{j=1}^{\nu}\frac{B_jC_{t,j}}{2A_j^2}\right)Q|Q|=0,\text{with } \delta_0=\int_0^L\frac{C_d}{2hA^2}dx+X \tag{3}$$

where $c = \int_0^L A^{-1}dx$, η is the free surface elevation (or head) difference between the ends of the channel and L is the length of the channel. Following [9] the coefficient $C_{t,j}$ has been introduced to 87 represent the thrust coefficient of an idealised turbine placed in the jth row of turbines, with the total area of turbines in the row blocking a fraction B_j of the local channel cross-sectional area A_j. The total number of rows of turbines is ν. The term X in (3) accounts for all losses proportional to the square of the channel flow rate, such as separation losses at the channel exit [5] and any abrupt changes in geometry.

To explore multiple tidal constituents, in this paper we represent the elevation difference across the channel as a series of sinusoidal components

$$\eta = a_0\cos(\omega_0 t)+a_1\cos(\omega_1 t)+\ldots+a_N\cos(\omega_N t). \tag{4}$$

For convenience, we non-dimensionalise equation (3) using.

$$Q' = \frac{ga_0}{\omega_0 c}Q,\ \lambda_0 = \frac{ga_0}{(\omega_0 c)^2}\delta_0,\ \lambda_1 = \frac{ga_0}{2(\omega_0 c)^2}\sum_{j=1}^{\nu}B_jC_{t,j},$$
$$t' = \frac{t}{\omega_0},\eta' = \frac{\eta}{a_0},k_n = \frac{a_n}{a_0},\Omega_n = \frac{\omega_n}{\omega_0}, \tag{5}$$

giving

$$\frac{dQ'}{dt'}+(\lambda_0+\lambda_1)Q'|Q'| = \cos(t')+k_1\cos(\Omega_1 t')+\cdots. \tag{6}$$

From equation (6) it is clear that the key non-dimensional parameter describing the natural dynamics in the channel is λ_0 [5]. This parameter describes the relationship between accelerating the mass (or inertia) of water in the channel and natural seabed

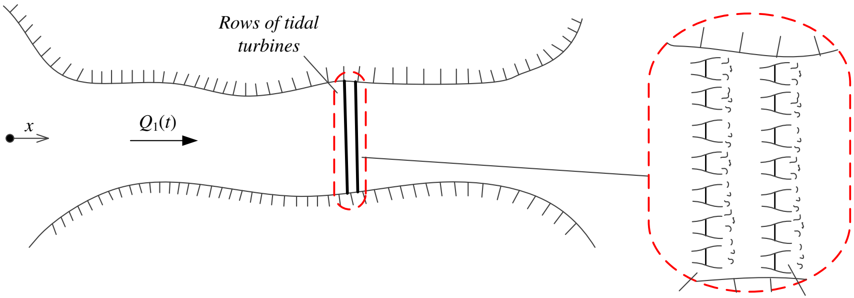

Fig. 1. Idealised tidal channel with rows of tidal turbines included (after Garrett and Cummins, 2005).