Soil Dynamics and Earthquake Engineering 114 (2018) 69–84

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

# The seismic displacement of a block sliding on an inclined plane with resistance varying with the distance moved

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ARTICLE INFO

*Keywords:*
Sliding-block model
Seismic displacement
Constitutive modeling
Strain softening
Slopes
Slip surfaces
Analytical solution

ABSTRACT

The paper considers a rigid block on an inclined plane. First, the typical response measured along slip surfaces of sands and clays under both drained and undrained conditions is identified. Then, for any expression simulating this change in shear stress with displacement along the boundary between the block and the plane, the paper derives an equation which predicts the equivalent friction angle which, when applied, computes the same seismic displacement of the block. The paper also proposes, validates and calibrates a simple constitutive equation which simulates the typical shear stress-displacement response measured in ring shear tests for both sands and clays sheared under both drained and undrained conditions. Next, the paper for the constitutive law above simulating the change in shear stress with displacement along the slip surface, derives equations which predict the equivalent friction angle which, when applied, computes the same seismic displacement of the block. Finally, based on these equations, the paper proposes an easy-to-apply methodology predicting the seismic displacement of slopes along slip surfaces with strain softening with the aid of the seismic displacement predicted by the conventional sliding-block model and applies this methodology in order (a) to validate it against results of elaborate numerical analyses and (b) to modify the Ambraseys-Menu expressions predicting the seismic displacement of the conventional sliding-block model for soils exhibiting strain softening.

## 1. Introduction

Engineers usually assess the seismic safety of slopes using the dynamic factor of safety calculated from loads. However, evaluations based on the dynamic factor of safety have the serious drawback that they do not consider the seismic displacement, which is directly related to damage [1]. Simplified models predicting the seismic displacements of slopes are less accurate than methods using finite-elements, but have the advantage of readily available solutions and of simplicity, which makes them usable by practicing engineers and for often-needed in soil mechanics extensive parametric analyses. Thus, further development of simplified models, such as the one presented in this paper, is appropriate. In addition, application of sophisticated finite-element models may by used to validate these simplified methods [2].

The "conventional" sliding-block model [3] forms the basis of simple models predicting permanent seismic shear displacement of soils [4]. As shown in Fig. 1, a rigid block rests on an inclined plane. The resistance along the block-inclined plane boundary follows the Mohr-Coulomb law. Critical acceleration is defined as the minimum horizontal acceleration which causes movement of the block. Every time where the applied horizontal acceleration is larger than the critical acceleration, the block slides. The total displacement of the block is obtained by the addition of the partial slips. Different empirical expressions have been proposed by different researchers, predicting the seismic displacement of the block in terms of its critical acceleration and characteristics of the applied seismic motion [4]. These solutions are used for the prediction of permanent seismic movement of slopes along a predefined slip surface, by appropriately selecting the equivalent slope and applied acceleration of the rigid block [4,5].

The conventional sliding-block model described above is generally successful in estimating small ground deformations without loss of strength. However, when the ground deformations are large, this model may not be accurate, primarily because of (a) loss of strength in soils along the slip surface [1], (b) changes of geometry of the sliding soil mass towards a gentler inclination [6,7] and (c) the dynamic response of the sliding mass [8]. The paper studies the effect (a) above. This

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https://doi.org/10.1016/j.soildyn.2018.06.028
Received 3 August 2017; Received in revised form 20 June 2018; Accepted 23 June 2018

**Nomenclature**

| | |
|---|---|
| A-M | Ambraseys-Menu |
| $a_1$, $u_1$, a, n | parameters of the constitutive Eq. (10) |
| $a_2$ | parameter given by Eq. (14) |
| $a_c$ | critical acceleration for relative motion of the sliding-block model |
| $a_{c\text{-}m}$ | critical acceleration for soil strength equal to $R_{res}/r$ |
| $a_{c\text{-}r}$ | critical acceleration for soil strength equal to $R_{res}$ |
| $a_{max}$ | the maximum value of the applied acceleration |
| $a(t)$ | applied acceleration history |
| $A_c$, $A_e$ | areas defined in Fig. 7a |
| b | factor given by Eq. (10c) |
| d | differential |
| Dr | relative density |
| Er | error defined by Eq. (17) |
| *Fc* | factor given by Eq. (21) |
| *G*, $G_{max}$ | shear modulus, maximum value of G |
| *g* | the acceleration of gravity |
| Id | factor given by Eq. (18a) |
| $l_{sb}$ | the length of the slip surface of the sliding-block model |
| $m_{sb}$ | mass of the sliding-block model |
| N, $F_N$ | normal and parallel force at the sliding-block model |
| OCR | overconsolidation ratio |
| *PI* | plasticity index (%) |
| $p'$ | effective octahedral stress |
| $P_f$ | the total (final) generated excess pore pressure |
| *Q*, *Rp* | parameters of Eq. (18a) |
| $R_o$, $R_{fin}$, $R_m$ | parameters defined by Eq. (19) |
| *R*, *R(u)* | $\tau/\sigma'_o$, R in terms of u |
| $R_{max}$, $R_{res}$, r | maximum R, final (residual) R, $R_{res}/R_{max}$ |
| *Ru* | factor defined by Eq. (16a) |
| *t* | time |
| *Stdev* | standard deviation |
| *u* | the displacement along the slip surface |
| $u'$ | the displacement given by Eq. (10d) |
| $u_f$, $u_a$ | the final value of u, the value of u where $d^2\tau/du^2 = 0$ |
| $u_o$ | the initial value of u given by Eq. (10e) |
| Wc | work defined by Eq. (6) |
| $\beta$ | the inclination of the sliding block |
| $\gamma_{cyc}$ | cyclic shear strain |
| $\sigma'_o$ | initial effective stress normal to the slip surface |
| $\tau$, $\tau_o$ | shear stress, the initial value $\tau$ |
| $\varphi_{sb}$ | the frictional resistance along the slip surface of the sliding-block model |
| $\varphi_{equiv}$ | the equivalent frictional resistance along the slip surface given by Eqs. (8), (20) |
| $\varphi'$ | the effective frictional resistance |

effect is very important, and building codes require its modeling. For example, Eurocode [9] states that "In modeling the mechanical behaviour of the soil media, the softening of the response with increasing strain level shall be taken into account."

Regarding the effect (a) above, ring shear tests illustrate that loss of strength in soils along slip surfaces is important (1) for dry dense sands [10], (2) for overconsolidated clays sheared under drained conditions [11], (3) for saturated sands, due to the build-up of excess pore pressures, which is considerable especially where grain crushing occurs [12], and (4) for clays sheared under undrained conditions, due to the build-up of excess pore pressures, which is considerable especially where a collapse of the soil structure occurs [13].

The effect (a) above has been modeled by different researchers using a rigid block with resistance along the slip surface of the block which follows constitutive equations of varying complexity [14–19]. In general, these constitutive equations are formulated in terms of effective stress, in order to predict not only the shear stress, but also the generation of excess pore pressure along slip surfaces under undrained conditions. The reason is that only these general formulations illustrate the difference in soil response under drained and undrained conditions, or as a result of the dissipation of these excess pore pressures during landslides [14–16,18,19].

Possibly the most complete constitutive equations predicting soil response along slip surfaces are those proposed for sands and clays by Stamatopoulos [16,19]. These equations, which have been thoroughly validated using an extensive number of ring shear tests, are based on (i) the proposition made by Aubry et al. [20] that the constitutive model of displacement and stress of an interface should be of the same form and derived from the constitutive model of strain and stress of continuum soils, (ii) the prediction of the soil response towards failure using the effective stress and critical state theories and (iii) the assumption that the critical state changes once failure is first reached, in terms of the applied further shear displacement, as a result of grain crushing in sands and re-direction of clay particles in clays. Fig. 2 presents typical shear stress-displacement response along slip surfaces, predicted by the models [16,19]. Shear stress-displacement response along slip surfaces of a dense sand and of an overconsolidated clay sheared under drained conditions and a sand exhibiting grain crushing and a clay exhibiting a collapse of the soil structure sheared under undrained conditions is simulated [11,21–23]. In all four cases the following response can be observed: First, the shear stress (τ) gradually increases from its initial value with shear displacement (u) at a decreasing rate, until the peak shear stress is reached. At the peak shear stress, the rate of change (dτ/du) is zero. Then, as u increases further, τ decreases to its residual value, while (dτ/du) (1) first decreases from zero until a limit displacement value reached, then (2) it starts to increase, and finally (3) at very large shear displacement it tends to zero.

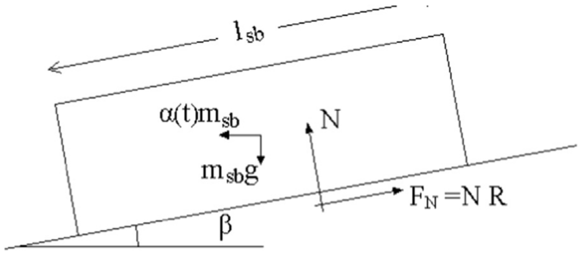

**Fig. 1.** A block sliding in an inclined plane with only frictional resistance. In the “conventional” sliding-block model, R = $\tan\varphi_{sb}$.

The objective of the paper is to propose an analytical method which predicts the seismic displacement along slip surfaces where soil resistance varies with shear displacement, utilizing predictions of the conventional sliding-block model. The following are needed and performed below to achieve this objective: (i) to collect from the relevant bibliography the soil response which has been measured along slip surfaces for both sands and clays sheared under both drained and undrained conditions, (ii) to consider a rigid block on an inclined slip surface and for any relevant to the response described in paragraph 6 above expression simulating the change in shear stress with displacement along the slip surface, derive equations which predict the equivalent friction angle which, when applied, computes the same seismic displacement of the block, (iii) to propose, calibrate and validate a constitutive model predicting the soil response along slip surfaces described in paragraph 6 above, which additionally produces an analytical solution of the procedure (ii), (iv) using the results of (ii), to derive an analytical solution giving the equivalent friction angle for resistance varying with shear displacement according to the