Soil Dynamics and Earthquake Engineering 114 (2018) 313–325

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Spectral representation-based dimension reduction for simulating multivariate non-stationary ground motions

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ARTICLE INFO

Keywords:
Earthquake ground motions
Multivariate non-stationary stochastic processes
Spectral representation
Dimension reduction
Random function
Strong motion records

ABSTRACT

A framework of spectral representation-based dimension reduction for simulating multivariate non-stationary stochastic ground motion processes is addressed in this paper. By means of introducing random functions serving as constraints correlating with the orthogonal random variables in the original spectral representation scheme, the high-dimensional randomness degree involved in the multivariate stochastic processes can thus be reduced substantially. To this aim, three random function forms considering the combination of trigonometric functions and orthogonal polynomials are constructed for simulation purpose. Accordingly, the accurate representation of the original stochastic processes is realized with merely three elementary random variables, overcoming the principal challenge of numerous random variables faced by the Monte Carlo simulation method. Also, the consistency of the statistics between the sample functions of stochastic ground motions and strong motion records is established with update of the ground motion model parameters in stochastic simulation techniques. Numerical investigations involving the comparisons with the Monte Carlo simulation method and the validation based on the strong motion records are presented to demonstrate the superiority and effectiveness of the proposed methodology in practical engineering applications.

1. Introduction

Being different from "point" structures, adequate attention should be paid to large-scale structures, bridges, dams, tunnels, buried pipelines and communication transmission systems, on which the effects of the spatial variability of earthquake ground motions are usually unignorable. Since the supports of large-scale structures would undergo differential movements during a severe earthquake event [1,2], completely representing the spatially variable ground motions, generally described as spatio-temporal stochastic fields through time and spatial variables, would become significantly important [3]. Nevertheless, in consideration of its characteristics of being non-repeatable, unevenly distributed in space, and dependent on local soil conditions, the recorded data of strong motions observed at dense instrument arrays, is thus far from enough for seismic response analysis and seismic reliability assessment with respect to large-scale engineering structures. Consequently, the Monte Carlo simulation method (MC scheme), applied to generate spatio-temporal stochastic fields as seismic inputs, has attracted marvelous attention from both the academic and the engineering communities and has been developed rapidly in recent years. In the families of the MC scheme, the spectral representation, owing to its accuracy and simplicity, appears to be the most versatile and also the most widely used one [4]. In enforcement, the multi-dimensional univariate ($m\mathrm{D}-1\mathrm{V}$) spatio-temporal stochastic fields (commonly known as the continuous form) are usually transformed into the one-dimensional multivariate ($1\mathrm{D}-n\mathrm{V}$) stochastic vector processes (commonly known as the discrete form) when simulating the spatially variable ground motions.

The spectral representation, also referred to as the wave superposition method, was first used by Rice to characterize a $1\mathrm{D}-1\mathrm{V}$stochastic noise in 1944 [5]. Though the concept of the spectral representation had been existed for a period of time, the real application of it for simulating $m\mathrm{D}-1\mathrm{V}$ or $m\mathrm{D}-n\mathrm{V}$ homogeneous or non-homogeneous stochastic processes was first proposed by Shinozuka in later years [6,7]. Since then, numerous significant contributions have been made by Yang [8], Shinozuka and Deodatis [9,10], Deodatis [11,12], Grigoriu [13], and Liang et al [14] in the latest forty years when the spectral representation developes fast. As in applications of simulating earthquake ground motions, Hao et al [15], utilizing a sequential approach and random phase variability in the simulations, first

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https://doi.org/10.1016/j.soildyn.2018.04.058
Received 5 March 2018; Received in revised form 14 April 2018; Accepted 29 April 2018
0267-7261/

presented a spectral decomposition simulation scheme to generate the spatially variable ground motions. Later, Deodaits [11] proposed a spectral representation-based simulation algorithm which could be used to simulate the multivariate non-stationary stochastic ground motions with evolutionary power. Since then, the spectral representation has been flourishing in simulating the multivariate non-stationary stochastic ground motions [16].

The spectral representation for simulating $1D - nV$ stochastic processes is based on spectral factorization through Cholesky decomposition and subsequent utilization of a summation of trigonometric series. However, being of huge expense in calculation memory and time is the main drawback of the spectral representation. Due to the simulation efficiency of the spectral representation is influenced by the superposition of trigonometric functions and decomposition of the matrix, related studies have been conducted by the following researchers. Yang [8] first employed the Fast Fourier Transform (FFT) algorithm technique to significantly enhance the superposition efficiency of trigonometric functions and proposed a formula to simulate random envelope processes. Cao et al [17] and Huang et al [18] respectively suggested a class of explicit expression for the Cholesky decomposition of the power spectral density (PSD) matrix.

To date, though the simulation efficiency of the spectral representation has been improved dramatically, the extremely high-dimensional randomness degree (the number of random variables) involved in the MC scheme still remains a principal challenge for it being applied in probability density evolution analysis and reliability assessment of large-scale structures [19]. The two main reasons, listed in the following, show us why the MC scheme is limited in engineering applications. The first one is the large number (always as many as thousands) of random samplings needed to present the time series of stochastic excitations. The second one is the incomplete expansion of stochastic excitations caused by the truncation in consideration of the calculation effort, which would then result in an inadequate quantification of probability propagation from the stochastic excitations to the dynamic responses of structures [20]. Thus, the way to efficiently reducing the randomness degree in the spectral representation has become a research hotspot recently. Bearing this situation in mind, Chen et al [21,22] developed the stochastic harmonic function representation of both stationary and non-stationary stochastic processes, which could obtain the accurate target PSD through a small number of random harmonic components. Meanwhile, Chen and Li [23] suggested the optimal determination of frequencies in the spectral representation of stochastic processes. Furthermore, Liu et al [20,24–26] proposed a dimension reduction approach by adopting random function in the spectral representation and Karhunen-Loeve expansion for simulating $1D - 1V$ stationary and non-stationary stochastic processes and $1D - nV$ (multivariate) stationary stochastic processes with only several elementary random variables. Moreover, another highlighted advantage of Liu's approach is that each representative sample generated by the proposed approach has definite probability information that enables it to be naturally combined with the probability density evolution method (PDEM) [19,27] to completely implement the dynamic response analysis and dynamic reliability assessment of engineering structures, especially suitable for the nonlinear and complicated cases. The application of Liu's approach has been performed on the probability density evolution analysis of a nonlinear concrete gravity dam subjected to non-stationary seismic ground motion [25].

Traditionally, the spectral representation contains two main schemes [28], i.e., the orthogonal-random-variables-based scheme (generally referred to as the random amplitudes formula) and the random-phase-angles-based scheme (generally referred to as the random phases formula), which are developed in parallel but in a dependent way. However, since the random-phase-angles-based scheme can take full advantage of the FFT algorithm technique to effectively speed up the calculation efficiency, and the number of random variables involved in it is half of that in the orthogonal-random-variables-based scheme, the random-phase-angles-based scheme thus possesses a relatively higher efficiency, making it more popular in engineering applications. As a matter of fact, there is an inseparable relation between the two schemes, which will be addressed in detailed statement in this study.

The preceding works mainly focused on the method simulating $1D - 1V$ non-stationary stochastic ground motions which only can be applied to the "point" structures, such as high-rise buildings, towers and chimneys with their supports set at local soil. As for the large-scale structures, taking multi-point seismic inputs into consideration, it is essential to bring forth the dimension reduction idea for simulating the non-stationary spatially variable ground motions. Therefore, this paper aims to extend the spectral representation-based dimension reduction to simulate the multivariate non-stationary stochastic ground motions. Further, while ensuring the computational efficiency and accuracy at the same time, it is hoped that the randomness degree and the number of sample functions will be minimized with the aid of the proposed method. Three random function forms, each of which combines the trigonometric functions and orthogonal polynomials, are constructed for the first time in order to achieve the simulation purpose. Benefitting from this proposed scheme, the high-dimensional randomness degree is efficiently reduced to merely three. Meanwhile, this study also establishes the consistency of the statistics between the representative samples of stochastic ground motions and the strong motion records, which provides a consummate reference for the stochastic simulation scheme being applied in engineering practices. The remaining contents of this paper are arranged as follows. Section 2 reviews the theory of spectral representation for multivariate non-stationary stochastic processes based on Cholesky decomposition of the coherence function matrix. The dimension reduction for simulating multivariate non-stationary stochastic processes is articulated in detail in Section 3. Section 4 lists the sample realization procedures for multivariate non-stationary stochastic processes. Numerical examples of multivariate non-stationary stochastic ground motions are shown in Section 5, and the comparisons with the MC scheme are also discussed in this section. In Section 6, the validation on the basis of strong motion records is presented. Conclusions drawn from this study are summarized in Section 7.

2. Spectral representation of multivariate non-stationary stochastic processes

Suppose that $\boldsymbol{X}(t) = \{X_1(t), X_2(t), ..., X_n(t)\}^T$ is a real-valued, zero-mean, $1D - nV$ non-stationary stochastic process, which can be represented as Fourier-Stieltjes integral given by [29]:

$$\boldsymbol{X}(t) = \int_{-\infty}^{\infty} \boldsymbol{A}(t, \omega) e^{i\omega t} d\boldsymbol{Z}(\omega) \tag{1}$$

where i is the imaginary unit, ω is the circular frequency and t is the time parameter. $\boldsymbol{A}(t, \omega)$ is supposed as a real-valued diagonal matrix, i.e., $\boldsymbol{A}(t, \omega) = diag\{A_1(t, \omega), A_2(t, \omega), ..., A_n(t, \omega)\}$, and its element $A_i(t, \omega)(i = 1, 2, ..., n)$ represents the intensity and frequency modulating function satisfying $A_i(t, -\omega) = A_i(t, \omega)$. $\boldsymbol{Z}(\omega)$ is an n-variate zero-mean complex vector process with its orthogonal increment satisfying the following basic conditions:

$$E[d\boldsymbol{Z}(\omega)] = \boldsymbol{0}_{n\times1}, \quad E[d\boldsymbol{Z}(\omega) d\boldsymbol{Z}^{*\mathrm{T}}(\omega')] = \delta_{\omega\omega'} \boldsymbol{S}(\omega) d\omega, \quad d\boldsymbol{Z}(-\omega) = d\boldsymbol{Z}^*(\omega) \tag{2}$$

where $E[\cdot]$ is the mathematical expectation. The superscripts * and T are complex conjugate and matrix transpose, respectively. $\delta_{\omega\omega'}$ is the Kronecker-delta. $\boldsymbol{S}(\omega)$ is the two-sided (stationary) PSD matrix.

The multivariate non-stationary stochastic process $\boldsymbol{X}(t)$ is generally presented through the correlation matrix and/or the evolutionary power spectrum density (EPSD) matrix. The correlation matrix is given as follows: