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Enhanced stochastic averaging of non-integrable nonlinear systems subjected to stochastic excitations

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ABSTRACT

Stochastic averaging method based on energy envelope for a non-integrable system reduces the dimension of dynamic systems, while preserving the nonlinearity in system behavior and its stochasticity arising from input excitations. However, this approach for a general system neglects effects of off-diagonal damping components, considers a lumped effect of diagonal damping entries, and assumes independent stochastic excitations for involved degrees of freedom. These constraints may limit the application of stochastic averaging as nonlinear systems often do not satisfy these conditions. This paper addresses these shortcomings and proposes a new strategy to derive an equivalent nonlinear stochastic system. For this purpose, an equivalent excitation intensity and modified damping parameters are derived by equating drift and diffusion components of the modified and the original system through the method of weighted residuals using high order moments of system velocities. The proposed approach, called enhanced stochastic averaging, is demonstrated for a one-story building on a raft foundation in loose sand subjected to lateral stochastic excitations. Results, such as probability density function at different intensity levels, indicate that the enhanced stochastic averaging method significantly improves the analytical predictions of the probabilistic properties of system responses compared to the conventional stochastic averaging method.

1. Introduction

Model characterization and response prediction of dynamic systems have posed challenges in many fields of science and engineering. Natural or engineered systems may exhibit nonlinear behavior in their dynamic response; these responses are often uncertain due to the stochasticity in input excitations and other sources. To generate simplified models for such systems, equivalent linearization techniques were developed [1]. This class of methods aims at determining parameters of an equivalent linear model by minimizing the error between the responses of the nonlinear and linear systems [2]. This technique can be stochastic, where the probabilistic nature of the excitation is considered, or deterministic, where the excitation is assumed to be known a priori. However, the accuracy of these methods reduces as the nonlinearity in the system and the number of the degrees of freedom (DOF) increase [2]. Stochastic averaging is an alternative solution that is proven to provide higher accuracy, since it retains the intrinsic nature of nonlinearity in the system behavior as well as effects of stochasticity in input excitations [3]. Stochastic averaging has been used for system identification and control design in different domains such as structural engineering [4–6], earth and environmental sciences [7], and physics [8].

Stochastic averaging can be used to derive approximate probabilistic solutions to problems involving lightly damped systems [9]. Such systems can be considered as diffusive Markovian processes with a transition probability density function (PDF) governed by the Fokker-Planck-Kolmogorov (FPK) equation. The drift and diffusion components of the FPK equation are derived by applying stochastic calculus. Subsequently, the PDF of system responses is computed by solving the FPK equation, and this can be used for response prediction of variables such as low and high order moments of energy, displacements, and velocities. In that regard, three methods of averaging have been proposed [9]: standard stochastic averaging, the averaging method of coefficients in FPK equation, and stochastic averaging of energy envelope. The first two methods were developed to derive the solution of FPK equation by averaging it with respect to time. The third method transforms the dynamic system into the Hamiltonian domain. This method can be applied to lightly damped systems subjected to weak excitations [9]. Additional features of stochastic averaging of energy envelope include lower computational demand and simpler parameter analysis compared to other stochastic averaging methods, since the dimension of the FPK differential equation is reduced. In stochastic averaging of energy envelope, the total energy of the system is known as the Hamiltonian.

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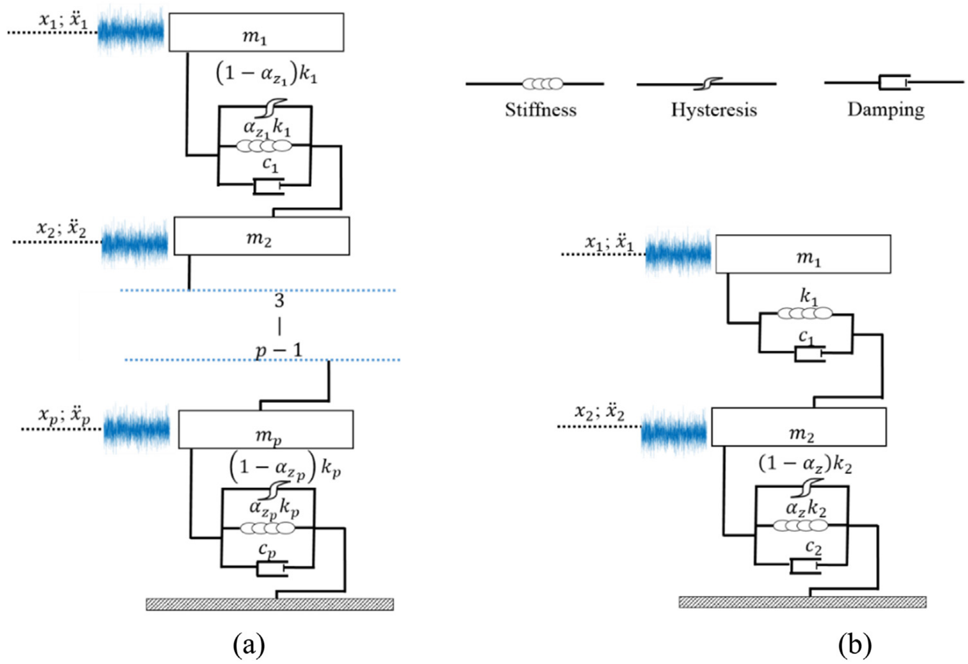

Fig. 1. Hysteretic systems: (a) multi-DOF hysteretic system and (b) 2-DOF hysteretic system in study.

Common response variables of dynamic systems e.g. displacements and velocities are often rapidly varying quantities, while energy envelopes of systems are slowly varying quantities. This method exploits this feature to average the rapidly varying processes to yield the averaged Itô equations for slowly varying processes.

In general, multi-DOF systems of interest are either integrable or non-integrable. Integrable systems are identified by uncoupled potential energies, where each DOF corresponds to an independent Hamiltonian. The integrable system is then averaged and the PDF of the Hamiltonian can be derived [10]. On the other hand, non-integrable systems include dependent potential energies, where the entire system can only be represented by a single Hamiltonian [11] for all DOFs. Stochastic averaging of energy envelope has been used for response prediction [12] and reliability estimation [13] for both integrable and non-integrable systems. For example, Zhu et al. [4] applied this method to the linear model of a tall building subjected to wind loadings that were simulated as filtered white noise. To simplify the analysis, the system was converted from a non-integrable system to an integrable system using linear modal analysis. The averaging was applied to the derived integrable system and a controller was designed accordingly. Stochastic averaging of energy envelope was also implemented for system representation of single DOF hysteretic systems subjected to white noise excitations [5]. This technique was further extended to support systems subjected to other types of excitations such as Poisson excitation [14].

Generally, real-world systems are non-integrable. Conventional stochastic averaging methods for such general systems neglect effects of off-diagonal damping terms, consider a collective contribution of the diagonal entries rather than their individual effects, and treat external stochastic excitations on applied DOFs as independent processes [15]. These factors may limit the application of stochastic averaging for nonlinear systems that often do not comply with those conditions. To address these limitations, this paper proposes a procedure to develop equivalent nonlinear models for hysteretic non-integrable multi-DOF systems subjected to stochastic excitations. In that respect, parameters of the equivalent nonlinear model including modified damping parameters and equivalent excitation intensity are derived through the application of the method of weighted residuals to the drift and diffusion components in the Itô equation for actual and modified systems. The proposed methodology, called enhanced stochastic averaging method, uses high order moments of velocity response variables to derive equivalent damping and intensity parameters; this improves the accuracy of the stochastic averaging method when applied to nonlinear non-integrable systems with coupled damping terms. The application of the new strategy is demonstrated for stochastic averaging of a nonlinear foundation-structure system subjected to stochastic excitations. Past stochastic analyses of structures have generally neglected contributions from soil and foundation behaviors by assuming fixed support conditions, or oversimplified nonlinear hysteretic behaviors of foundations by using linear models. This study considers a one-story building on a nonlinear raft foundation in loose sand subjected to Gaussian white noise excitations. The probability density function of the Hamiltonian and its moments are derived using the proposed method and are compared to those computed using conventional stochastic averaging and Monte Carlo simulations (MCS). It should be noted that the proposed method is general and can be applied to multi-DOF non-integrable hysteretic systems. The rest of the paper is organized as follows. Section (2) presents a comprehensive derivation of the proposed methodology. Next, results of the numerical study are presented in Section (3), and conclusions of the research are summarized in Section (4).

2. Methodology

Natural and engineered dynamic systems are often nonlinear and non-integrable. A multi-story building supported by a nonlinear foundation subjected to lateral excitations is among such systems. A representative model for these structures is shown in Fig. 1a where the DOFs of the system are coupled through damping and stiffness over the entire building. In conventional stochastic averaging methods, effects of the off-diagonal damping terms vanish and a collective contribution rather than individual effects of diagonal terms is considered. In addition, these methods treat external excitations on DOFs of systems as independent processes, while in many cases such as the structure in Fig. 1a, these disturbances are perfectly correlated, or have high degrees of correlation. These limitations may result in unacceptable response predictions of realistic systems using conventional methods of stochastic averaging of energy envelope. In this paper, these conventional strategies are extended to provide more accurate representation