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Natural frequencies of piled raft foundation including superstructure effect

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ABSTRACT

Dynamic characteristics of piled raft foundation system plays an important role in the safety of high-rise buildings subjected to seismic loadings though the analytical study considering the effect of both foundation and superstructure is very few in literature. The present study first proposes an exact analytical solution for piled raft foundation subjected to harmonic excitation and resting on an elastic Winkler foundation to obtain its natural radial frequency. After successful validation through available centrifuge test results, a series of parametric study has been carried out investigating the influence of various geometrical and geotechnical parameters of the foundation and the soils respectively. It is observed that the pile length and the pile diameter has significant effect on the natural radial frequency of the foundation system whereas soil density and spacing between piles have minimal effect. The importance of stiffness of the superstructure is also considered in the proposed methodology. It is found that the natural radial frequency of piled raft foundation including superstructure stiffness decreases by 12% to 28% when compared with the computation of the natural radial frequency excluding effect of superstructure stiffness. Hence, this study provides a new analytical methodology to obtain the dynamics characteristics of piled raft foundation considering the superstructure effect which can be used for the design.

1. Introduction

Since few decades, piled raft foundation is well-known as an economic foundation solution where piles are installed beneath the raft considering serviceability requirement of structure [1,2]. Its successful use for several high rise buildings can be found in Germany, Japan, UK, UAE and many other countries [3–7]. In seismic regions, such buildings are also subjected to dynamic loads containing a wide range of frequencies. An existing pseudo-static method of analysis does not consider the influence of natural frequency of the system in calculating its dynamic response. During earthquake, these structures are subjected to seismic loads having varying frequencies which needs severe attention because if the natural frequency of the system comes closer to the predominant frequency of an earthquake excitation, massive failure can be expected. Several researchers have shown interest investigating theses responses under seismic loading conditions. Gazetas and Dobry (1984) [8] first studied the response of a single pile and a pile group using Winkler modeling approach. Horikoshi et al. (2003) [9] performed centrifuge study to obtain the behavior of piled raft foundation subjected to dynamic loading. Matsumoto et al. (2004) [10] carried out shaking table tests on pile-raft model embedded in dry sand by varying the height of the superstructure. Baziar et al. (2018) [11] studied influence of superstructure on seismic behavior of piled raft foundation by using centrifuge modeling. Kumar et al. (2016) [5] and few other researchers [12,13] carried out numerical study to investigate the behavior of piled raft foundation under dynamic loadings. However, an analytical methodology to quantify the dynamic characteristics of the pile-raft-superstructure system in terms of natural frequency have not been investigated till date. Here, an exact analytical solution is proposed for a piled raft system resting on the elastic Winkler foundation. A parametric study is then carried out to investigate the influence on the natural frequency of foundation system with changes in its geometrical as well as stiffness parameters. Later, the effect of stiffness of the superstructure is also considered in the proposed new methodology and its influence on the natural frequency of the system is investigated.

2. Methodology of modeling piled raft foundation

Piles are assumed to behave as Euler's beam element resting on the linear Winkler foundation where lateral soil springs and dampers are

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List of notations

The following symbols are used in this paper:

A	cross sectional area of pile
D_r	relative density
d	pile diameter
E_o	modulus of deformation
E	elastic modulus
G	shear modulus
I	moment of inertia
k_{ps}	pile-soil stiffness coefficient
k_{rs}	raft-soil stiffness coefficient
L	pile length
W_r	total vertical load on raft including self-weight
y	lateral displacement of pile at each node
y_r	lateral displacement of raft
ρ	density of reinforced concrete
μ	Poisson's ratio of soil
λ	predominant radial frequency of earthquake
ω	natural radial frequency of the system
ξ	damping ratio of soil

attached along with their lengths. Fig. 1 illustrates a piled raft foundation configuration idealized to single piled raft unit attached to lateral springs and dampers considering Kelvin-Voigt model. Considering the equilibrium of a pile element, the governing differential equation of a pile based on free vibration analysis can be written as:

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} + k_{ps}dy + \eta\frac{\partial y}{\partial t} = 0 \quad (1)$$

where y is the lateral displacement of the pile in m, x is the distance along the pile starting from top in m, η is the damping coefficient of soil in kN-s/m, k_{ps} is the modulus of subgrade reaction in MN/m^3 calculated by using Equation 2 [14], ρ is the density of reinforced concrete, A is the cross-sectional area of the pile, EI is the Flexural rigidity of the pile in kN m^2.

$$k_{ps} = 80E_0d^{-0.75} \quad (2)$$

where $E_O = 0.7$ N, N is the SPT-N value, E_O is the modulus of deformation in MN/m^2, d is the diameter of the pile in the cm. The differential equation for the raft based on a free vibration of a rigid body can be expressed as:

$$m\frac{d^2y_r}{dt^2} + k_{rs}y_r + R + \eta\frac{dy_r}{dt} = 0 \quad (3)$$

where m is the total mass of the raft including the load coming from the superstructure in kg, y is the displacement of the raft in meter, k_{rs} is the horizontal raft-soil stiffness modulus in kN/m^2 calculated by using Equation 4 [15] and R is the shear force acting between pile and raft in kN.

$$k_{rs} = \frac{32(1-\mu)Ga}{7-8\mu} \quad (4)$$

where G is the shear modulus of soil in kN/m^2 obtained from the $E = 2G(1+\mu)$, where Young's modulus, $E = 0.5(N + 15)$, N is SPT-N value of soil [16], a is the equivalent radius of a raft element in meter, μ is the Poisson's ratio of the soil. The condition of compatibility is maintained at junction between raft and pile head. A solution of the differential equation of a pile can be assumed as:

$$y = z(x)q(t) \quad (5)$$

where $q(t)$ is a function of natural radial frequency ω, {e.g. $q(t) = Qe^{i\omega t}$}. Inserting this form of solution into the differential Equation, following form is obtained:

$$\left[\frac{\partial^4 z}{\partial x^4} - \left(\frac{\omega^2}{c^2} - \frac{k_{ps}d}{EI}\right)z + \frac{i\eta\omega z}{EI}\right]q = 0;\ c = \sqrt{\frac{EI}{\rho A}} \quad (6)$$

For solving the Eq. (6), the real part can be expressed as:

$$\frac{\partial^4 z}{\partial x^4} - \beta^4 z = 0;\ \beta^4 = \frac{\omega^2}{c^2} - \frac{k_{ps}d}{EI} \quad (7)$$

The general solution of the governing differential Equation is given as:

$$z = c_1(\cos\beta x + \cosh\beta x) + c_2(\cos\beta x - \cosh\beta x) + c_3(\sin\beta x + \sinh\beta x) + c_4(\sin\beta x - \sinh\beta x) \quad (8)$$

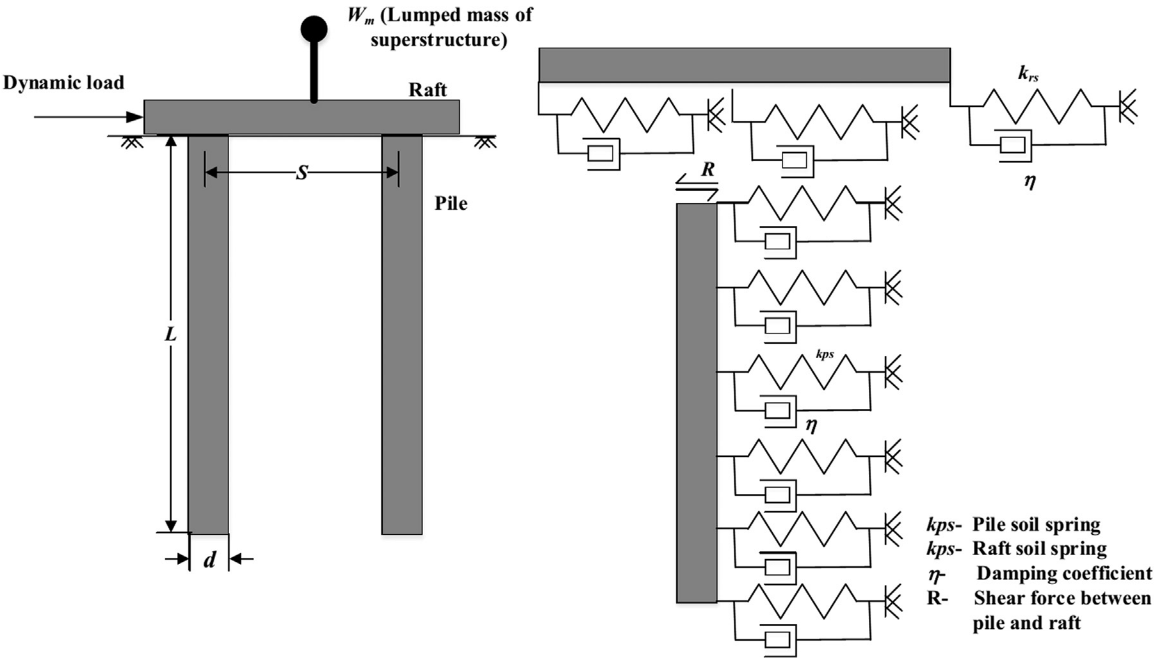

Fig. 1. Idealization of piled raft foundation system into single pile-raft unit.