Soil Dynamics and Earthquake Engineering 112 (2018) 198–202

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Improved equivalent mass-spring model for seismic response analysis of two-dimensional soil strata

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ARTICLE INFO

Keywords:
Seismic analysis
Mode equivalence
MDOF system
Mass-spring model
Amplification

ABSTRACT

This paper presents an improved development of the mass-spring model initially proposed by Okamoto and Tamura for seismic response analysis of two-dimensional soil strata. The new model is based on the mode equivalence method and is constructed as a two-dimensional equivalent multi-degree-of-freedom (MDOF) system. Physical parameters of the model are derived and formulated in terms of modal properties. Dynamic response of the model from inclined SH-waves is obtained and the accuracy and validity are confirmed by comparing with the solution of wave propagation. The improved model is further discussed through a performance comparison with the mass-spring model.

1. Introduction

In 1973, Okamoto and Tamura [1] proposed a mass-spring model method for seismic response analysis of immersed tunnels based on dynamic model tests and earthquake observations. Although these tests and the resulting method appear to be simpler than the most recent ones [9-12], the formers gave the first insight into the features of the seismic response of immersed tunnels. According to this method, a mass-spring model is developed to represent the soil strata along the tunnel axis. As depicted in Fig. 1a, the surface layer above the base rock is divided into a number of soil slices, perpendicular to the tunnel axis. Each slice is represented by an equivalent mass-spring system that consists of a lumped mass, a spring and a dashpot. The neighboring masses are connected to each another by springs and dashpots along the tunnel axis to simulate the connection between the adjacent soil slices. With this model, the response of the surface layer can be calculated by the dynamic equilibrium equation [2].

However, the equivalent mass-spring system oversimplifies the ground response by only accounting for the fundamental shear vibration of the represented soil slice [3]. Moreover, the inertia center of the soil slice in the fundamental mode is used as the location of the lumped mass to determine the stiffnesses of the longitudinal springs between adjacent masses [4]. This is inconsistent with the statement that the lumped mass of an equivalent single-degree-of-freedom (SDOF) system is located at the fundamental modal effective height [5]. In addition, the longitudinal springs (Fig. 1b) represent the resistance to the axial or shear relative displacement between adjacent soil masses and are determined in accordance with different deformation modes of ground vibration [3]. However, it is unclear whether the determination method is reasonable. As a result, the mass-spring model may lead to an unreliable ground response.

This paper aims to develop a new equivalent discrete model for seismic response analysis of two-dimensional (2D) soil strata by improving the mass-spring model. The one-dimensional (1D) equivalent multi-degree-of-freedom (MDOF) system is adopted to substitute the equivalent SDOF system that represents the soil slice. The physical parameters of the 1D equivalent MDOF system are formulated in terms of the prescribed modal properties. Then a series of 1D equivalent MDOF systems are spring connected at corresponding masses such that the overall soil strata is constructed into a 2D MDOF system having multiple lumped masses in both the longitudinal and vertical directions. The longitudinal springs are evaluated by using the mode equivalence principle instead of resisting the relative deformation as in the mass-spring model. Dynamic response of the 2D equivalent MDOF system from inclined SH-waves is derived and compared with the theoretical solutions based on 2D wave propagation to validate its accuracy. Moreover, the mass-spring model is reevaluated through a comparative analysis with the 2D equivalent MDOF system.

2. One-dimensional equivalent MDOF system

The mass-spring model is based on the assumption that the ground

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https://doi.org/10.1016/j.soildyn.2018.05.001
Received 30 April 2018; Accepted 1 May 2018

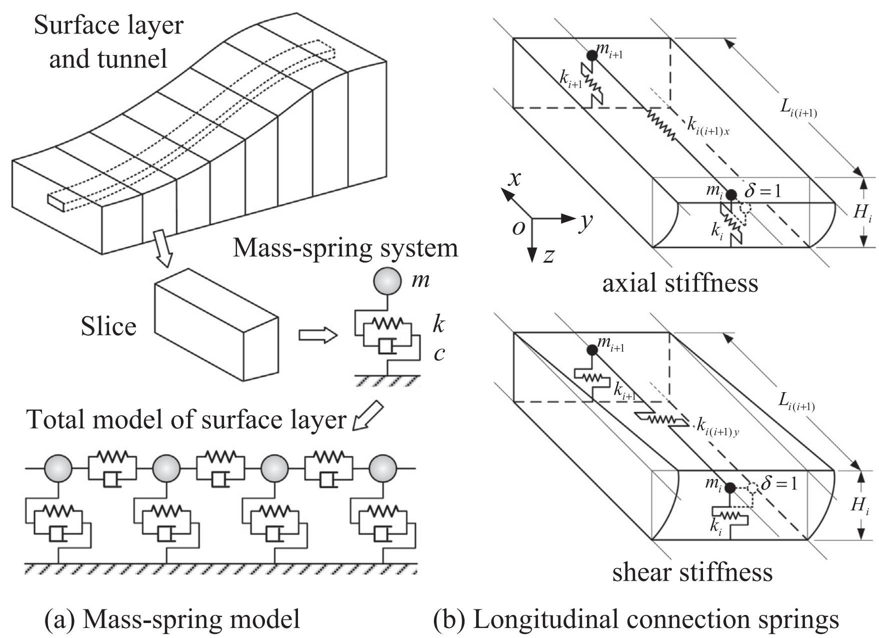

Fig. 1. Mass-spring model and determination of the longitudinal connection springs
Adapted from Kiyomiya [2] and Okamoto et al. [3].

response is dominated by the fundamental shearing vibration and hence the higher modes are neglected. In order to consider more modes of ground response, Li et al. [6] generalized the mode equivalence method from the SDOF system to the MDOF system and developed a 1D equivalent MDOF system to represent the 1D soil column. The basic principle of the mode equivalence method requires that the modal properties of the equivalent system coincide with those of the original system in corresponding modes. According to the extended mode equivalence method, four mode-normalization-independent modal properties, i.e. the natural frequency, the modal effective mass, the modal effective height and the modal damping ratio, are used to determine the physical parameters of the equivalent system. A detailed description of the derivation process for the 1D equivalent MDOF system can refer to [7]. Herein the necessary physical parameters are provided for the convenience of the following derivation of the 2D equivalent MDOF system.

For a 1D equivalent system of J DOFs, the mass $(m_j)_1^J$ and spring $(k_j)_1^J$ constants are expressed as

$$m_j = \frac{\Theta_{j-1}^2}{\Psi_{j-1}\Psi_j} \tag{1}$$

$$k_j = \frac{\Theta_{j-1}\Theta_j}{\Psi_j^2} \tag{2}$$

where

$$\Theta_j = \sum_{l=1}^{C_J^{J-j}} \left(\prod_{n\in \mathbf{S}_l\binom{J-j}{J}}^{J-j} M_n^e \omega_n^2 \right)\left(\prod_{m<n\in \mathbf{S}_l\binom{J-j}{J}}^{C_{J-j}^2} (\omega_m^2-\omega_n^2)^2 \right) \tag{3}$$

$$\Psi_j = \sum_{l=1}^{C_J^{J-j}} \left(\prod_{n\in \mathbf{S}_l\binom{J-j}{J}}^{J-j} M_n^e \omega_n^4 \right)\left(\prod_{m<n\in \mathbf{S}_l\binom{J-j}{J}}^{C_{J-j}^2} (\omega_m^2-\omega_n^2)^2 \right) \tag{4}$$

$\mathbf{S}_l\binom{J-j}{J}$ is the lth number set formed by arbitrarily selecting $J-j$ numbers from the set of integers from 1 to J; ω_n and M_n^e are the nth natural frequency and the nth modal effective mass of 1D soil column.

The height of the jth mass over the base is given by

$$h_j = \sum_{n=1}^{J} \varphi_{jn}\Gamma_n h_n^e \tag{5}$$

where φ_{jn} and Γ_n are the nth mode shape of the jth mass and the nth modal participation factor of the 1D equivalent MDOF system; h_n^e is the nth modal effective height of 1D soil column.

3. Two-dimensional equivalent MDOF system

In attempt to build the 2D equivalent MDOF system by the mode equivalence method, the soil strata under investigation need to be confined in an extent in the horizontal direction since only bounded soil strata has 2D modes. Consider the 2D soil strata with a finite length of $2L$ and a total thickness of H in the Cartesian coordinate system as depicted in Fig. 2a. The coordinates x, y and z denote the longitudinal, transverse and vertical directions, respectively. Besides the free surface and the fixed base as the 1D soil column, both side boundaries of the 2D soil strata in the horizontal direction are free. Similar to the mass-spring model, the 2D equivalent MDOF system is constructed by longitudinally connecting a series of 1D equivalent MDOF systems at corresponding masses as shown in Fig. 2b. The longitudinal springs between neighboring 1D equivalent MDOF systems are to be determined.

Free vibration of the $2P \times J$-DOF system in Fig. 2b is governed by

$$\boldsymbol{m}\ddot{\boldsymbol{u}} + \boldsymbol{k}\boldsymbol{u} = \boldsymbol{0} \tag{6}$$

The mass and stiffness matrices, owing to the axis symmetry of the system with respect to $x = 0$, can be expressed in a partitioned form as

$$\boldsymbol{m} = \mathrm{diag}(\boldsymbol{m}_P,\ \ldots, \boldsymbol{m}_2,\ \boldsymbol{m}_1,\ \boldsymbol{m}_1,\ \boldsymbol{m}_2,\ \ldots, \boldsymbol{m}_P) \tag{7}$$

$$\boldsymbol{k} = \mathrm{symm{-}tridiag}\begin{pmatrix} \boldsymbol{k}_P\ ,\ \cdots\ ,\ \boldsymbol{k}_2\ ,\ \boldsymbol{k}_1\ ,\ \boldsymbol{k}_1\ ,\ \boldsymbol{k}_2\ ,\ \cdots\ ,\ \boldsymbol{k}_P \\ \boldsymbol{k}_{(P-1)P}, \ldots, \boldsymbol{k}_{12},\ \boldsymbol{k}_{01},\ \boldsymbol{k}_{12}, \ldots, \boldsymbol{k}_{(P-1)P} \end{pmatrix} \tag{8}$$

where

$$\boldsymbol{m}_p = \mathrm{diag}(m_{p,1},\ m_{p,2},\ \ldots, m_{p,J}) \qquad (p = 1,\ 2,\ \ldots, P) \tag{9}$$