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Dynamic analysis of axially loaded end-bearing pile in a homogeneous viscoelastic soil

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ABSTRACT

A new approximate analysis technique is developed to obtain the steady-state response of axially loaded pile embedded in a homogeneous, isotropic soil, and resting on a rigid base. In the analysis, the soil is modeled as an axisymmetric linear viscoelastic continuum with frequency independent hysteretic material damping and the pile is modeled as an elastic Euler rod with a circular cross-section. The soil displacement in the vertical direction is expressed as a product of separable functions, while the radial displacement is assumed to be zero for mathematical simplicity; although the effect of the radial displacement is indirectly taken into account by modifying the soil modulus. The Extended Hamilton's principle in conjunction with the calculus of variations is used to obtain the differential equations governing pile and soil displacements and the relevant boundary conditions. The pile and soil displacement equations are solved analytically following an iterative algorithm. The accuracy of the analysis is ensured by comparing the pile responses obtained from this analysis with those obtained by other methods available in the literature. A parametric study is performed to investigate the influence of the pile and soil parameters on the axial dynamic pile-head impedances. The plots developed from the parametric study can be used in the design.

1. Introduction

Structures like tall buildings, bridges, transmission towers, oil and gas platforms, and wind turbines often have piles as foundations that are subjected to dynamic axial loads from machine vibrations, traffic, and earthquakes. An interest in the analysis of such axially loaded pile foundations lies in the prediction of appropriate dynamic pile-head impedance for different frequencies of the harmonic axial load applied to the pile head. The impedance function is a complex quantity in which the real part quantifies the axial stiffness of the pile-soil system and the imaginary part quantifies energy dissipation from both radiation damping and material damping [1,2]. This complex impedance function can be used to evaluate the dynamic response of the superstructure by representing the pile-soil system by an axial spring and a dashpot connected in parallel and characterized by the real and imaginary parts of the impedance functions, respectively.

Several methods are reported in the literature for the dynamic analysis of axially loaded piles embedded in a homogeneous or layered soil with each soil layer being isotropic or transversely isotropic, either overlying a rigid base or floating in a half-space. These studies can be grouped into four categories based on their rigour and analysis techniques: (i) Winkler based analytical and numerical formulations [3–10], (ii) rigorous, three-dimensional (3-D) continuum-based formulations with numerical solutions using finite element (FE), boundary element (BE) or mixed FE-BE methods [11–19], (iii) rigorous 3-D analytical or semi-analytical, continuum-based studies [20–28], and (iv) approximate analytical continuum-based studies [29–32]. Of the different analysis methods available, the Winkler based formulations are the most popular and widely used by geotechnical engineers because these approaches are computationally inexpensive and have the ability to incorporate soil layering and nonlinear soil behavior. However, the methods based on Winkler approach require parameter calibration for accurate prediction of pile and superstructure response and either neglect the coupled vibration between the pile-soil or between the soil layers. The rigorous continuum-based numerical solutions (e.g., finite element methods) have an advantage over the Winkler type formulations as these methods consider the coupled vibration of the pile and soil. However, these methods are computationally intensive and expensive, and may also require the modeling of non-reflecting viscous boundary conditions to include the effect of radiation damping which can affect the accuracy of the solution. The rigorous analytical or semi-analytical continuum-based studies have the advantage of taking into account the effect of material and radiation damping within the formulation and solution process; however, the mathematics involved are

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often complex, computationally expensive and not quite useful for routine geotechnical practice. The advantage of the approximate analytical continuum-based methods is that these methods have the ability to capture the important aspect of the physics of the problem without being mathematically too complex and computationally too intensive.

In this paper, a new approximate analytical continuum-based method for dynamic analysis of an axially loaded single pile with circular cross-section embedded in a homogeneous, isotropic soil, and resting on a rigid base is developed. The analysis assumes the pile as a Euler rod and the soil as an axisymmetric linear viscoelastic continuum with frequency independent material damping. In the analysis, the equilibrium of the pile-soil system is considered using the Extended Hamilton's principle, and the differential equations governing the pile and soil displacements are obtained using the calculus of variations. The differential equations of pile and soil displacements are coupled and the solution is obtained analytically following an iterative algorithm. The accuracy of the analysis technique is validated by comparing the complex dynamic pile-head impedances obtained from this analysis with those obtained from an approximate and a rigorous analytical solution technique available in the literature. A parametric study is performed to investigate the influence of different pile-soil parameters on the dynamic axial response of piles. The advantage of the analysis technique is that it is mathematically simple yet rigorous and the solution can be obtained very quickly.

2. Analysis

2.1. Problem definition

A pile with a circular cross-section modeled as an elastic rod of radius r_p, length L_p, Young's modulus E_p, and density ρ_p is considered to be embedded in a soil layer overlying a rigid base (Fig. 1). The soil layer is modeled as a continuum, that is homogeneous, isotropic, and linear viscoelastic with hysteretic material damping [33] characterized by density ρ_s and complex Lame's constant $\lambda_s^*=\lambda_s(1+2j\xi_s)$ and $G_s^*=G_s(1+2j\xi_s)$ where $\lambda_s = E_s\nu_s/\{(1+\nu_s)(1-2\nu_s)\}$, $G_s = E_s/\{2(1+\nu_s)\}$, E_s is Young's modulus, ν_s is the Poisson's ratio, ξ_s is the frequency independent damping ratio of soil, and $j=\sqrt{-1}$. The pile head is subjected to a time-harmonic vertical force $V(t) = V_0 e^{j\Omega t}$ (see Fig. 1) where Ω = circular forcing frequency, V_0 = forcing amplitude, and t = time. The objective of the analysis is to obtain the steady-state pile head displacement and the vertical dynamic pile-head impedances. In the analysis, no slippage or separation between the pile and the surrounding soil is considered. A right-handed cylindrical (r-θ-z) coordinate system is chosen for the analysis such that its origin lies at the center of the pile head and the z-axis coincides with the pile axis which points downward with the angular distance θ measured clockwise positive.

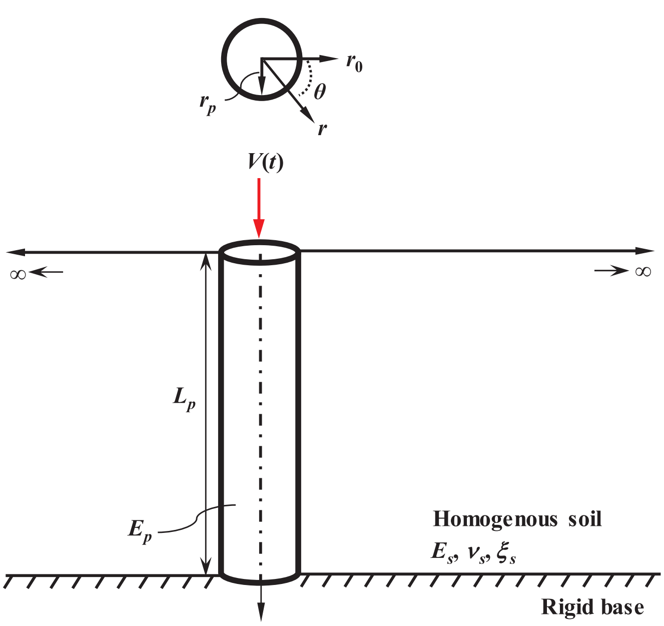

Fig. 1. Axially loaded pile embedded in a homogeneous, isotropic, viscoelastic soil, and resting on a rigid base.

2.2. Soil displacement, stress-strain, strain-displacement and strain energy density

As the problem is axisymmetric, only the radial and vertical displacements u_r and u_z are non-zero and are functions of r, z, and t. Often, radial soil displacement is assumed to be small and neglected in the analysis of axially loaded piles [1,31,32]. The assumption of zero radial displacement ($u_r = 0$) is made in this analysis as well, and the vertical soil displacement u_z is mathematically expressed as [34,35]

$$u_z(r, z, t) = w(z, t)\phi(r) \tag{1}$$

where $w(z,t) = w(z)e^{j\Omega t}$ where $w(z)$ is the steady-state axial pile displacement, $\phi(r)$ is the dimensionless displacement function assumed to be equal to one at $r = r_p$ (this ensures perfect contact between pile and soil at the pile-soil interface) and equal to zero at $r = \infty$ (this ensures that soil displacements induced by the loaded pile decrease with increase in radial distance from the pile and eventually becomes zero at large radial distance).

The stress-strain relationship of the homogeneous, isotropic, and linear viscoelastic soil for the axisymmetric problem is written as

$$\begin{bmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\\ \sigma_{zz}\\ \sigma_{rz}\end{bmatrix} = \begin{bmatrix}\lambda_s^*+2G_s^* & \lambda_s^* & \lambda_s^* & 0\\ \lambda_s^* & \lambda_s^*+2G_s^* & \lambda_s^* & 0\\ \lambda_s^* & \lambda_s^* & \lambda_s^*+2G_s^* & 0\\ 0 & 0 & 0 & G_s^*\end{bmatrix}\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\\ \gamma_{rz}\end{bmatrix} \tag{2}$$

The strain-displacement relationship (with contractive strains assumed positive) is obtained by substituting Eq. (1) in the strain vector on the right-hand side of Eq. (2) and is given by

$$\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\\ \gamma_{rz}\end{bmatrix} = \begin{bmatrix}-\frac{\partial u_r}{\partial r}\\ -\frac{u_r}{r}-\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\\ -\frac{\partial u_z}{\partial z}\\ -\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)\end{bmatrix} = \begin{bmatrix}0\\ 0\\ -\phi(r)\frac{\partial w(z,t)}{\partial z}\\ -w(z,t)\frac{\partial \phi(r)}{\partial r}\end{bmatrix} \tag{3}$$

Substituting the Eq. (3) in Eq. (2) and on simplification, the strain energy density $\sigma_{pq}\varepsilon_{pq}/2$ (σ_{pq} and ε_{pq} are soil stress and strain tensors and summation is implied by the repetition of the indices p and q) of soil is obtained as

$$U_D = \frac{1}{2}\left[(\lambda_s^*+2G_s^*)\left(\frac{\partial w}{\partial z}\right)^2\phi^2 + G_s^* w^2\left(\frac{\partial \phi}{\partial r}\right)^2\right] \tag{4}$$

Eq. (4) is derived with the assumption that soil displacement in the radial direction is zero. The effect of radial displacement on the response of axially loaded pile is small; nevertheless, neglecting the radial displacement completely introduces artificial restraint in the pile-soil system. This restraint can be reduced by replacing $(\lambda_s^*+2G_s^*)$ on the right-hand side of Eq. (4) with $\eta_s G_s^*$ where $\eta_s = 2/(1-\nu_s)$. A similar approximation was made by Mylonakis [31] and Anoyatis and