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A substructure replacement technique for the numerical solution of wave scattering problem

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ABSTRACT

This paper is devoted to the numerical solution of wave scattering and diffraction problems. A substructure replacement technique is proposed for this purpose. By introducing an arbitrary auxiliary boundary, the scattering problem is solved as a simple radiation one. The proposed approach is effective for wave scattering and diffraction by surface irregularities, such as hills and valleys as well as by subsurface inhomogeneities, such as cavities, tunnels and pipelines buried in backfilled trenches. Irrespective of how complex the different types of surface irregularities and subsurface inhomogeneities are, the same input and the same treatment can be applied. It is equally applied to P, SV, SH and Rayleigh wave incidence. Results are validated by comparison with those available in the literature. A challenging test for rather higher frequency scattering of SH wave from a symmetrical shallow V-shaped canyon has been carried out. Well agreements have been achieved. The proposed technique is simple in theory and easy to implement numerically.

1. Introduction

Problems related to wave scattering and diffraction by surface and subsurface topographies and inhomogeneities, such as valleys, canyons, foundation of structures, cavities, pipes, subways and tunnels are of great importance in civil Engineering. It may be of interest in the study of the site effects induced by local amplification of strong ground motion, which can be significant over a large frequency interval. The noisy impact on the environmental buildings from underground urban railways is of increasing public concern. Wave scattering is also significant and sometimes a critical aspect to be considered for the dynamic soil-structure interaction analysis of infrastructures including buildings, bridges, subways, industrial units, dams, power plants and so on.

Since the initial work of Trifunac in 1973 [1] on two dimensional response of semi-circular canyon subjected to SH-wave excitation, significant progress concerning the wave scattering and diffraction has been made in the past decades. A comprehensive literature review of available methods may be found in [2–6], an extensive bibliography is included in [2,6]. In the following, for brevity, only a few significant ones are presented for reference. Essentially, closed-form analytical solutions are worked out for simple geometrics, which can give a deep physical insight on the nature of diffracted waves and which offer the best benchmarks to test and to verify other approximate solutions. The advance of modern computational techniques makes numerical approaches more feasible for realistic engineering problems.

The wave function method (WFM) and the boundary integral equation (BIE) method or boundary element method (BEM) have gained relatively wide spread popularity. The former utilizes the expansion of wave functions, which satisfy the equation of motion and radiation condition at infinity. However, in general they do not satisfy traction free boundary condition on the surface of the half-space, which should be imposed locally. The SH wave scattering can be solved by imaging method to ease the treatment of boundary condition on the surface of the half-space [1,7]. Lee and Liu proposed a proper form of the orthogonal cylindrical-wave functions for both the longitudinal P- and SV- waves [8] such that they can simultaneously satisfy the zero-stress boundary conditions at the half-space surface. The solution of the wave function method is obtained by truncating an infinite system of equations in the infinite number of unknown coefficients [9–12]. Most of the articles has been concentrated on two-dimensional models. Three-dimensional cases are limited to axisymmetric geometries [13]. A few on non-axisymmetric canyons [14].

The advantage of the BIE method or BEM is obvious. The discretization is applied only to the boundaries of the scatters, leading to significantly reduction of the number of unknown variables and the radiation condition at infinity is satisfied rigorously. However, the evaluation of fundamental solutions is needed, which often requires a large amount of computational effort. The BIE method includes both

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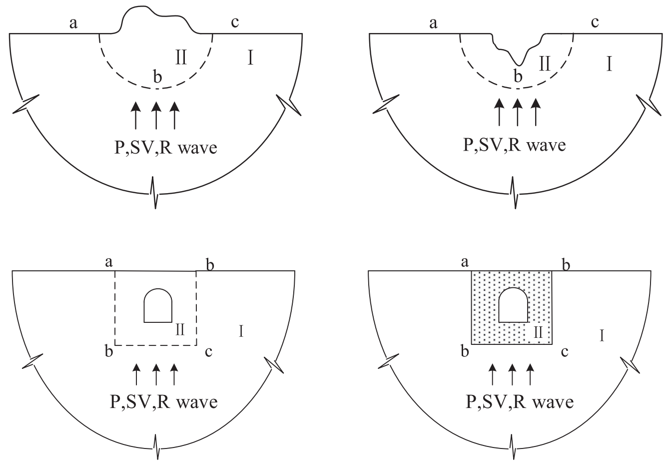

Fig. 1. wave scattering by free-surface relief and sub-surface inclusion.

the direct ones and the indirect ones. In the former formulation, the unknown variables are the sought values of displacements and tractions [15,16], while in the later formulation, a distribution of forces is searched so that the radiated field satisfies the boundary conditions, and then the displacements are obtained by superposition of the radiation from these sources [17–21].

Attempts have also been made to combine integral equations with discrete wavenumber expansions of Green's functions [22,23]. In general, BIE methods are not suitable for analyzing inhomogenous media. On the other hand, the finite element (FE) and finite difference (FD) methods are versatile in handling a medium with varying material properties. This prompted development of the hybrid methods, which combines FEM with the BIE method [25], or combines FEM with the wave expansion technique [24]. Hybrid methods exploit the versatility of FEM for detailed modelling of the near field and the effectiveness of the BIE methods or the wave function expansion technique in the far field.

A complex variable function method developed by Muskhelishvili [26] for the solution of problems in elasticity has also been applied to deal with the scattering problems. Following the idea and the expressions proposed in [27] for dynamic response analysis of irregularly-shaped cavity, scattering of plane P, SV and Rayleigh waves by a shallow lined tunnel in an elastic half-space has been solved [28]. Applying conformal mapping technique the shallow subsurface lined tunnel in the physical domain is transformed into two concentric annuli onto the complex variable domain. The boundary value problem results in a set of infinite algebraic equations for the determination of the coefficients of the series of wave potentials.

Recent publications show a trend to extend previous theories and methods to study wave scattering and diffractions in more realistic circumstances, or more complex engineering cases. In [29,30] the direct boundary element method is applied to simulate wave fields that develop in a isotropic half-plane containing different types of heterogeneities such as free-surface relief, unlined and lined tunnels, as well as multiple buried inclusions. In [31] zero stress, cylindrical wave functions are derived around a circular underground tunnel in a flat, elastic half-space subjected to P-wave incidence. In [32] a hybrid BEM-FEM technique is presented that reveals different effects of the seismic response of a complex soil-structure system and takes into account the lateral inhomogeneity of the layered geological region and the wave path from the source till the soil environment of the structure. In [33] scattering of plane harmonic P, SV or Rayleigh waves by a completely embedded corrugated cavity is studied using a direct boundary integral equation method. In [34,35] by applying region-matching technique and choosing a suitable auxiliary boundary the wave function analytic approach can be extended to blend stress singularities into the solution procedure, and scattering of SH waves by a shallow V-shaped canyon and a circular sectorial canyon have been studied.

In this paper a novel approach, for the solution of wave scattering and diffraction is proposed. By introducing an arbitrary auxiliary boundary in the half-space, the problem is reduced to a radiation one. Then the scattered displacement and stress fields induced by free-surface relief and sub-surface inclusions are solved as the dynamic response of the substructure. Furthermore the free field ground motion along the auxiliary boundary is used as input, thus the solution is greatly simplified. It can be equally applied to study the disturbance induced by SH, P, SV and Rayleigh waves. The proposed approach is simple in theory and easy to implement numerically. Numerical examples for various cases of wave scattering are provided and the results are compared with those available in the literature to validate the applicability and accuracy of the proposed technique.

2. Formulation of the basic equations

The idea of substructure replacement technique is addressed. Consider wave scattering phenomena by free-surface relief and sub-surface inclusions as shown in Fig. 1. Through introducing an arbitrary auxiliary boundary abc (or abcd), the half-space is divided into two parts, the far field regionIand the near-field regionII. The far-field regionI is solved as a radiation problem. Once the dynamic-stiffness matrix of the soil $[S_{bb}^{g}]$ and the corresponding motion $\{u_b^g\}$ along the auxiliary boundary abc (or abcd) are evaluated, then the dynamic response of the substructure, i.e. the near-field regionII, including the scattering field by free-surface relief or sub-surface inclusion can be readily determined. Accordingly, the scattering problem is solved as a simple radiation one. And exactly the same $[S_{bb}^{g}]$, $\{u_b^g\}$ can be equally applied to study the scattering field induced by different type of free-surface relief, sub-surface inclusion as well as multiple inclusions of arbitrary shape. We noticed that Luco in 1986 [36] studied the relation between radiation and scattering problems.

Furthermore, refer to Wolf [37] the free-field ground motion $\{\tilde{u}_b^f\}$ is used to replace $\{u_b^g\}$. As $\{\tilde{u}_b^f\}$ does not depend on the excavation, the evaluation of $\{\tilde{u}_b^f\}$ should be much easier than the evaluation of $\{u_b^g\}$. The superscript wavelet of $\{\tilde{u}_b^f\}$ is introduced here to denote the free-field motion, where excavated part is replaced by thy half-space soil (Fig. 2). In the following, a more detailed derivation in addressed.

The equation of motion of the substructure (regionII) is written as

$$\begin{bmatrix} [S_{ss}] & [S_{sb}] \\ [S_{bs}] & [S_{bb}] \end{bmatrix} \begin{Bmatrix} \{u_s^t\} \\ \{u_b^t\} \end{Bmatrix} = \begin{Bmatrix} \{F_s\} \\ -\{R_b\} \end{Bmatrix} \quad (1)$$

where $[S]$ denotes the dynamic stiffness, $[S_{ss}]$, $[S_{sb}]$, $[S_{bs}]$ and $[S_{bb}]$ are the stiffness matrices of the substructure; $\{u_s^t\}$ denotes the amplitudes of the total displacement; $\{F_s\}$ denotes the external load acting on the substructure; $\{R_b\}$ denotes the interaction force between the substructure and the half-space. The subscripts b and s denote the degrees of freedom lying on the boundary abc (or abcd) and the remaining part of the substructure, respectively. For problems of wave scattering in half-space the dynamic stiffness of the system is frequency dependent, so we formulate the equation of motion in the frequency domain. In this way, the dynamic-stiffness matrix [S] of the substructure abs (or abcd) is evaluated by low frequency expansion as follows

$$[S(\omega)] = [K](1 + 2\zeta i) - \omega^2[M] \quad (2)$$