Soil Dynamics and Earthquake Engineering 113 (2018) 442–453

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Vertical and torsional vibrations of a rigid circular disc on a transversely isotropic and layered half-space with imperfect interfaces

Ernian Pan[a,*], Heng Liu[b], Zhiqing Zhang[a,c]

[a] Dept. of Civil Engineering and Dept. of Mathematics, University of Akron, USA
[b] School of Transportation & Logistics, Dalian University of Technology, Dalian, China
[c] College of Urban Construction, Zhejiang Shuren University, Hangzhou, China

ARTICLE INFO

Keywords:
Soil-structure interaction
Transverse isotropy
Layered half-space
Rigid circular disc
Integral least-square
Imperfect interface
Vertical and torsional vibration

ABSTRACT

Rigid disc vibration over a layered half-space is essential to understand soil-structure interaction (SSI) between a foundation and the structure over it. In this article, we present a novel semi-analytical and very efficient method to calculate the SSI coefficients on the layered foundation due to both vertical and torsional vibrations. The material properties in each layer are transversely isotropic to take care of the possible different Young's moduli in different directions (vertical vs. horizontal). Furthermore, the interface between the layers is assumed to be general so that possible loose bonding between the layers (where the displacements and/or tractions are discontinuous across the interface) can be considered. The semi-analytical solution is based on the recently developed forward solution of the layered structure with imperfect interface under time-harmonic loadings within the circle on the surface. Since the present SSI problem is a mixed boundary-value problem, the method of superposition in terms of the powerful cylindrical system of vector functions combining with the integral least-square approach is proposed. To take care of multiple layers, the dual variable and position (DVP) is further introduced. The cylindrical system of vector functions has the advantage of separating the torsional vibration from the vertical vibration and yet expressing the two types of solutions uniformly. After validating the proposed semi-analytical method, numerical examples are presented to demonstrate the effect of material layering, interface imperfection, elastic anisotropy, and input frequency on the SSI coefficients, and on the surface displacement and stress variation. These new results should be also good benchmarks for future numerical methods.

1. Introduction

Soil-structure interaction (SSI) is essential in the analysis of dynamic response of structures over soil/rock foundations [1,2]. In terms of mathematical formulation, one needs to solve a mixed boundary-value problem between the structure (usually assumed as a rigid disc) and the half-space. By assuming that the half-space is homogeneous isotropic elastic, various analytical solutions were derived so far [3–9]. Furthermore, for the torsional vibration, analytical solutions were also obtained for certain special variation of the shear modulus vs. depth [10–12]. For the more general case, however, the problem has to be solved numerically [13]. In the past, various numerical and semi-analytical methods have been proposed to solve the SSI problems. Numerically, these include the finite-element method [14], boundary-element method [15], and their combination [16]. Two types of semi-analytical approaches have been also proposed to solve some related and simplified problems. One is the integral equation method (i.e., Luco and Westmann [17]) and the other is the semi-approximation method (i.e., Lin et al. [18]). We mentioned that under static deformation, the potential function method was also proposed to analyze the indentation experiments [19–21], and that using the analytical layer element and global matrix method, Ai et al. [22–25] solved both the two-dimensional and three-dimensional vertical vibration problems.

Recently, an efficient and semi-analytical forward method was proposed to study the dynamic response of a layered half-space under time-harmonic loading on the surface [26]. It is based on the dual variable and position (DVP) method in terms of the cylindrical system of vector functions. While the DVP method was proposed to handle problems associated with high frequency, thin layer, material anisotropy, the cylindrical system of vector functions was introduced to take care of the general loading cases with yet obvious physical correlation (LM-type and N-type correspond to the vertical and torsional deformations respectively). In this paper, we apply this forward method to solve the present mixed-boundary value problem where we have a rigid circular disc under vertical and torsional vibrations which rests on a transversely isotropic and layered half-space with imperfect interfaces

* Corresponding author.
E-mail address: pan2@uakron.edu (E. Pan).

https://doi.org/10.1016/j.soildyn.2018.06.014
Received 16 February 2018; Received in revised form 8 May 2018; Accepted 9 June 2018

(i.e., the interface across which the displacements and tractions are not continuous). Instead of using the numerical methods, such as the finite element method or boundary element method, we develop the following efficient and accurate semi-analytical method: We first solve the forward problem multiple times for the time-harmonic loads applied over different annular ring areas on the surface of the half-space in terms of the mathematically beautiful cylindrical system of vector functions and the DVP method; We then superpose these solutions and enforce them via an integral least-square to satisfy the mixed boundary conditions. This approach has been demonstrated to be very accurate and efficient for the corresponding static vertical indentation case [27]. Here in this paper, we extend this semi-analytical approach to the corresponding vertical and torsional vibrations and demonstrate that it is equally accurate and efficient in solving the present SSI problems. The remaining part of this paper is organized as follows. In Section 2, we describe the mixed boundary-value problem. We then introduce the novel forward-based semi-analytical method in Section 3. After validating our new method, various numerical examples are presented and discussed in Section 4, and conclusions are drawn in Section 5. Appendix A lists the basic forward formulation for both vertical and torsional vibrations in terms of the cylindrical system of vector functions and the DVP method.

2. Description of the mixed boundary-value problem

Fig. 1 shows a massless rigid circular disc of radius R over a transversely isotropic and layered half-space. The disc is under a time-harmonic vibration in vertical $Ve^{i\omega t}$ or hoop (torsional $Te^{i\omega t}$) direction. The layered half-space is made of n layers over a homogeneous elastic or rigid half-space. A Cartesian coordinate system $(x_1, x_2, x_3) = (x, y, z)$ is attached to the layered half-space with its origin at the center of the disc and on the surface of the layered half-space. The layered half-space is in the positive z region with layer j being bonded by its upper interface z_{j-1} and lower interface z_j with thickness $h_j = z_j - z_{j-1}$. Interfaces between different layers can be perfectly bonded or in imperfect connection (like interface at $z = z_j$). The disc is assumed to be well-bonded to the surface of the layered half-space without friction. The goal of this paper is to find, under the time-harmonic loads V and T (proportional to $e^{i\omega t}$), the response of the layered half-space (on the surface) below the disc, i.e., the structure-soil interaction (SSI) coefficients δ_V and α_T (proportional to $e^{i\omega t}$). Listed below is the detailed description of this mixed boundary-value problem.

2.1. Basic equations in each layer

Each layer is assumed to be transversely isotropic with its axis-symmetry being along the z-axis direction. The constitutive relation between the stresses σ_{ij} and displacements u_i in terms of the cylindrical system can be written as

$$\begin{aligned}
\sigma_{rr} &= c_{11}u_{r,r} + c_{12}r^{-1}(u_{\theta,\theta} + u_r) + c_{13}u_{z,z}\\
\sigma_{\theta\theta} &= c_{12}u_{r,r} + c_{11}r^{-1}(u_{\theta,\theta} + u_r) + c_{13}u_{z,z}\\
\sigma_{zz} &= c_{13}u_{r,r} + c_{13}r^{-1}(u_{\theta,\theta} + u_r) + c_{33}u_{z,z}\\
\sigma_{\theta z} &= c_{44}(u_{\theta,z} + r^{-1}u_{z,\theta});\ \sigma_{rz} = c_{44}(u_{z,r} + u_{r,z})\\
\sigma_{r\theta} &= c_{66}(r^{-1}u_{r,\theta} + u_{\theta,r} - r^{-1}u_\theta)
\end{aligned} \tag{1}$$

In Eq. (1), subscripts ",i" ($i = r,\theta,z$) denotes the derivative with respect to the coordinate variable, c_{ij} are the elastic stiffness constants with $c_{66} = (c_{11}\text{-}c_{12})/2$. As in the common practice, to represent the weak damping in the elastic materials, all the elastic properties c_{kj} are perturbed away from their real values by multiplying a small material damping factor β in the numerical calculation later on (i.e., c_{kj} are replaced by $c_{kj}(1 + 2\beta\mathrm{i})$ with i being the imaginary number) (i.e., Chen [28]). It is also noted that for a transversely isotropic material layer, there are a total of five independent elastic constants, which can be further expressed in terms of five engineering coefficients as

$$\begin{aligned}
c_{11} &= \frac{E_h[1-(E_h/E_v)\nu_v^2]}{(1+\nu_h)[1-\nu_h-(2E_h/E_v)\nu_v^2]}\\
c_{12} &= \frac{E_h[\nu_h+(E_h/E_v)\nu_v^2]}{(1+\nu_h)[1-\nu_h-(2E_h/E_v)\nu_v^2]}\\
c_{13} &= \frac{E_h\nu_v}{1-\nu_h-(2E_h/E_v)\nu_v^2};\quad c_{33} = \frac{E_v(1-\nu_h)}{1-\nu_h-(2E_h/E_v)\nu_v^2}\\
c_{44} &= \mu_v;\quad c_{66} \equiv \frac{c_{11}-c_{12}}{2} = \frac{E_h}{2(1+\nu_h)} = \mu_h
\end{aligned} \tag{2}$$

where E_h and E_v are the Young's moduli; ν_h and ν_v Poisson's ratios; and μ_h and μ_v are the shear moduli [29].

The time-harmonic equations of motion in the frequency domain with angular frequency ω $(=2\pi f)$ in each layer are (i.e., the stresses and displacements are all proportional to $e^{i\omega t}$)

$$\begin{aligned}
&\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}(\sigma_{rr}-\sigma_{\theta\theta}) + \rho\omega^2 u_r = 0\\
&\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} + \rho\omega^2 u_\theta = 0\\
&\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{rz} + \rho\omega^2 u_z = 0
\end{aligned} \tag{3}$$

where ρ is the density of the layer.

2.2. Mixed boundary conditions on the surface

It is noted that all the field quantities should vanish at infinity for the applied time-harmonic load (V or T) on the disc. The mixed boundary condition on $z = 0$ (with frictionless contact) between the disc of radius R and the surface of the half-space (under both vertical and torsional loads) is (in terms of the cylindrical coordinates (r,θ,z) and proportional to $e^{i\omega t}$):

$$\begin{aligned}
&\textit{Vertical}: \begin{cases} u_z(r,0) = \delta_V, & 0 \le r \le R;\\ \sigma_{zz}(r,0) = 0, & r > R;\\ \sigma_{rz}(r,0) = 0, & r \ge 0;\end{cases}\\
&\textit{Torsional}: \begin{cases} \theta_z(r,0) = \alpha_T, & 0 \le r \le R;\\ \sigma_{\theta z}(r,0) = 0, & r > R;\end{cases}
\end{aligned} \tag{4}$$

where δ_v and α_T are the unknown vertical displacement and angle of rotation on surface of the layered half-space, induced by the given vertical V and torsional T vibrational load, respectively. It should be further pointed out that the solution corresponding to V vibration is axis-symmetric, and that the solution to T vibration has only nonzero displacement component u_θ and shear stress component $\sigma_{\theta z}$. In other words, the shear stress (σ_{rz}) is zero for vertical vibration and the normal stress is zero for torsional vibration ([30,31]).

On the bottom of the layered half-space at $z = z_n$, depending if the last homogeneous half-space is rigid or elastic, the conditions there will be different. Under a time-harmonic vertical load on the surface, this

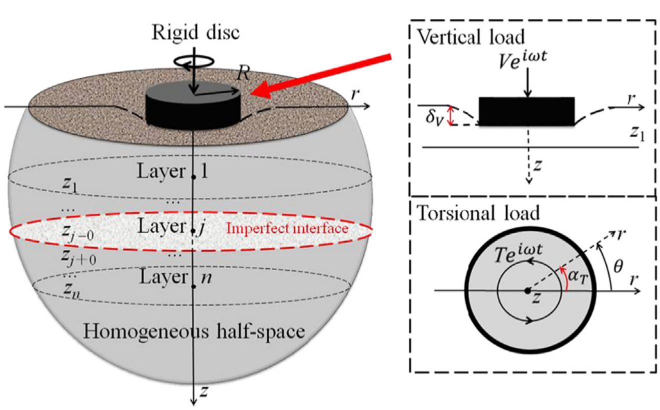

Fig. 1. Time-harmonic vibration of a massless rigid circular disc on a transversely isotropic and layered half-space with imperfect interfaces induced by the applied time-harmonic load $Ve^{i\omega t}$ or $Te^{i\omega t}$ on it.