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Seismic response analysis of nonlinear structures with uncertain parameters under stochastic ground motions

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ABSTRACT

An orthogonal expansion of stochastic ground motion model is incorporated into the probability density evolution method (PDEM). In this regard, a new methodology for seismic response analysis of nonlinear structures with uncertain parameters under stochastic ground motions can be formulated. The fundamentals and numerical algorithms of the methodology are introduced. Then, a rotational quasi-symmetrical point strategy (RQ-SPS) is developed for selecting the representative points in the random-variate space, which is of paramount importance to the tradeoff of accuracy and efficiency for the proposed methodology. An eight storey shear frame structure under seismic excitations, which exhibits strong nonlinear mechanical behaviors, is investigated. The effectiveness of the RQ-SPS is first verified, where the results by Monte Carlo simulations are also provided for comparisons. Then, case studies are implemented, in which the randomness involved in both stochastic ground motions and structural parameters are taken into account. The computational results demonstrate that the effect of randomness in structural parameters cannot be ignored compared to that in stochastic ground motions. Some features of the PDF evolutionary process of response are also discussed.

1. Introduction

Performance-based seismic design or control is of paramount importance for engineers to design earthquake-resilient structures, where various parameters involved in seismic ground motions and structural properties should be treated as uncertain quantities [1,2]. On the other hand, it is almost inevitable that the engineering structures will experience nonlinearity when subjected to disastrous earthquakes [3–6]. In this regard, the randomness and nonlinearity need to be taken into account simultaneously for the performance-based seismic design [7], which provides a comprehensive understanding of structural behaviors under seismic excitations.

Traditionally, the randomness is dealt with separately in stochastic dynamics of structures [8]. When the randomness in the modeling of structures is considered, the so-called stochastic finite element method (SFEM) or the random structural analysis [9,10] is established. Various approaches are available for calculating the response variability in the context of SFEM. With the development of SFEM, a variety of approaches such as the random perturbation technique [11], the path integral technique [12–15] and the orthogonal polynomial expansion method [9,16,17] have been well developed. However, great difficulties arise even for obtaining the second-order statistics of response of strongly nonlinear structures under seismic excitations [18]. Although the Monte Carlo simulation(MCS) [19,20] or its variants [21–23] is versatile regardless of nonlinearity, the computational effort is always intractable in practical engineering. On the other hand, the method treating the randomness involved in seismic excitations is referred to as the random vibration method. Extensive developments in the random vibration method such as the pseudo-excitation method [24], the equivalent linearization method [25], the stochastic nonlinear equivalent method [26], the FPK equation method [27–29] and the Hamiltonian method [30], etc. have been well investigated. Unfortunately, the solution to multiple-degree-of-freedom (MDOF) nonlinear structures subjected to random seismic excitations is still an open challenge. To authors' knowledge, the investigation of stochastic dynamics considering the randomness in both structural properties and external seismic excitations, which is also called the compound random vibration, is rather limited. The MCS method seems to be the only feasible method under this circumstance if the prohibitively large computational burden is ignored. However, it is widely accepted as a checking method for verification of a newly developed method.

Recently, a new method named probability density evolution method (PDEM) has been well developed by Li and Chen [31,1,32,33] for nonlinear stochastic dynamic problems. This method treats the

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randomness on a unified basis by invoking the random event description of the principle of preservation of probability and the embedded physical mechanism [8]. In this regard, the SFEM problems, the random vibration problems and the compound random vibration problems can be tackled in a unified framework by PDEM. Besides, the PDEM is capable of deriving the instantaneous probability density function (PDF), which is an intrinsic description of random dynamic systems; hence all the necessary information of the interested random dynamic system, e.g. statistical moments, reliability, etc. can be obtained without difficulty. It is therefore possible to conduct seismic response analysis of complex engineering structures taking into account the randomness in both ground motions and structural parameters for the performance-based design or control.

The objective of the present paper is to develop a new methodology based on PDEM to carry out seismic response analysis of structures considering the randomness involved in both ground motions and structural parameters. The paper is arranged as follows. In Section 2, the stochastic ground motion model based on the orthogonal decomposition is first introduced. Section 3 devotes to providing the fundamentals of PDEM and its numerical algorithms, where the determination of representative points plays an important role in achieving the tradeoff of accuracy and efficiency. In Section 4, a new strategy is proposed for selecting the representative points in PDEM to conduct repeated deterministic dynamic response analyses. Numerical example is investigated to validate the effectiveness of the proposed methodology and case studies are also implemented in Section 4. Some features of the responses are observed and discussed. Concluding remarks are included in the final section.

2. Stochastic ground motion model

Since Housner [34] first described earthquake ground motions as stochastic processes, extensive methods have been developed for rationally describing and modeling the stochastic seismic excitations. This research has spawned the development of methods rooted in two categories: the physical modeling and the mathematical expansion. In the physical modeling, the physical mechanism of an earthquake is incorporated to formulate the explicit expression of the random function, where the basic random variables are determined by the observed real data [35–39]. As for the mathematical expansion, the methods such as Karhunen-Loeve (KL) decomposition [40,41], the spectral representation method (SRM) [42–48], the stochastic harmonic function method [49,50] and the orthogonal decomposition method(ODM) [51], etc. have been well studied. As is known, a large number of random variables could be involved in the KL decomposition and the SRM, which may lead to significant numerical errors or infeasible computational efforts. In the present paper, the orthogonal decomposition method is specifically adopted to model the non-stationary seismic ground stochastic process because it is feasible to represent the stochastic process with only a few of random variables.

2.1. Orthogonal expansion of stochastic processes

Consider a real-valued stochastic process, which is denoted as $X(\varpi, t)$, with a zero mean and a finite second order moment. It is known that the stochastic process $X(\varpi, t)$ can be expanded by orthogonal expansion such that [51,52]

$$X(\varpi, t) = \sum_{k=1}^{N} \gamma_k(\varpi)\phi_k(t) \tag{1}$$

where the symbol ϖ represents the random nature of the corresponding quantity and $\{\phi_k(t), k = 1, 2, \ldots, \infty\}$ is a set of orthogonal base functions satisfying

$$\langle \phi_k, \phi_l \rangle = \int_0^T \phi_k(t)\phi_l(t)dt = \delta_{kl} \tag{2}$$

where δ_{kl} denotes the Kronecker's delta, i.e.

$$\delta_{kl} = \begin{cases} 1, & k = l \\ 0, & otherwise \end{cases} \tag{3}$$

Further, the correlated random vector $\mathbf{\Gamma} = [\gamma_1(\varpi), \gamma_2(\varpi), \ldots, \gamma_N(\varpi)]^T$ can be transformed to be the function of a set of standard uncorrelated random variables $\{\xi_j(\varpi), j = 1, 2, \ldots N\}$ such that

$$\mathbf{\Gamma} = \sum_{j=1}^{N} \mathbf{\Phi}_j \sqrt{\lambda_j}\xi_j(\varpi) \tag{4}$$

where λ_j s and $\mathbf{\Phi}_j$ s are the eigenvalues and standard eigenvectors of the correlation matrix $\mathbf{R}$, i.e. $\mathbf{R}\mathbf{\Phi}_j = \lambda_j\mathbf{\Phi}_j$ and

$$\mathbf{R} = [\rho_{ij}]_{N\times N} \tag{5}$$

where the correlation coefficient ρ_{ij} is

$$\rho_{ij} = \int_0^T \int_0^T R_X(\tau)\phi_i(t_1)\phi_j(t_2)dt_1dt_2, \qquad i, j = 0, 1, \ldots, (N-1) \tag{6}$$

where T is the time duration and $R_X(\tau) = R_X(t_1, t_2)$ is the auto-covariance function.

In this regard, the random variable $\gamma_k(\varpi)$ can also be written as

$$\gamma_k(\varpi) = \sum_{j=1}^{N} \sqrt{\lambda_j}\xi_j(\varpi)\varphi_{j,k} \tag{7}$$

where $\varphi_{j,k}$ denotes the k-th element of $\mathbf{\Phi}_j$.

Then, the stochastic process can be approximated by

$$X(\varpi, t) = \sum_{k=1}^{N}\sum_{j=1}^{N} \sqrt{\lambda_j}\xi_j(\varpi)\varphi_{j,k}\phi_k(t) = \sum_{j=1}^{N} \sqrt{\lambda_j}\xi_j(\varpi)f_j(t) \tag{8}$$

where

$$f_j(t) = \sum_{k=1}^{N} \varphi_{j,k}\phi_k(t) \tag{9}$$

It is obvious that the functions $\{f_j(t), j = 1, 2, \ldots, N\}$ are orthogonal with each other with the time domain, i.e.

$$\langle f_i(t), f_j(t) \rangle = \int_0^T f_i(t)f_j(t)dt = \delta_{ij} \tag{10}$$

The series expansion in Eq. (8) is therefore referred to as the orthogonal expansion of a stochastic process. In many practical applications, the eigenvalues λ_j may quickly decrease to be zero with the increase of j, which means a large number of correlated random variables can be represented by a small number of uncorrelated ones. Thus, Eq. (8) can be reduced to be

$$X(\varpi, t) \approx \sum_{j=1}^{r} \sqrt{\lambda_j}\xi_j(\varpi)f_j(t) \tag{11}$$

where $r \ll N$.

It is seen that the key issue is to determine the orthogonal base functions to efficiently implement the orthogonal expansion of a stochastic process. In this paper, the Hartley basis function is specifically adopted, i.e.

$$\phi_k(t) = \left(\sqrt{\frac{2}{T}}\right) cas\left(\frac{2k\pi t}{T}\right) \tag{12}$$

where

$$cas(t) = \cos(t) + \sin(t) \tag{13}$$

2.2. Orthogonal expansion of non-stationary seismic ground motions

Generally, the non-stationary seismic acceleration stochastic process can be expressed as [51]