Soil Dynamics and Earthquake Engineering 111 (2018) 8–13

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Determination of damping ratios for soils using bender element tests

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ARTICLE INFO

Keywords:
Bender element
Damping ratio
Soil
S-wave
Hilbert transform

ABSTRACT

The use of bender elements to measure small-strain soil properties in the laboratory has become increasingly accessible in recent years. Coupled with the fact that bender elements can be incorporated into conventional test apparatuses such as the triaxial test brings about great savings in time and resources as the bender element tests can be conducted concurrently with conventional geotechnical test for the same soil specimen. While primary (P) and shear (S) waves in bender element tests can provide soil stiffnesses reliably, damping ratio of the soil was seldom determined. This paper attempts to show numerically and experimentally that it is possible to determine the damping ratio of soils by applying the Hilbert transform method (HTM) to the bender element test results.

1. Introduction

Determination of small-strain soil properties using bender element tests is becoming more common. One great advantage of bender elements is that bender elements can be incorporated into existing apparatuses such as the triaxial test apparatus. To date, bender elements test is frequently used to determine small-strain stiffness but not damping ratio.

The objective of this paper is to investigate the feasibility of determining damping ratio from bender element tests, numerically and experimentally using the Hilbert transform method (HTM). First, bender element tests were simulated using finite element modelling and the damping ratio determined using HTM. Second, results of bender element tests on standard Ottawa 20–30 sand were used to obtain its damping ratio under various confining pressures. The damping ratios were compared with those from the literature obtained using the resonant column apparatus.

2. Determination of damping ratio

There are several methods of determining damping ratio from a seismic trace. One of the first used in the field of geophysics is the spectral ratio method (SRM) [1]. The SRM when applied to bender element tests has two configurations and involves the comparisons of the signals obtained from two specimens. The first configuration makes use of a reference specimen of similar dimensions and known damping ratio [2–5]. The second configuration involves using two identical specimens but of different heights [6–8]. The damping ratio can be obtained from either configuration using the equation below:

$$\ln\left(\frac{A_1}{A_2}\right) = \ln\left(\frac{GF_1}{GF_2}\right) + \left(\frac{\pi x_2}{Q_2 V_2} - \frac{\pi x_1}{Q_1 V_1}\right) f \tag{1}$$

where subscripts 1 and 2 represents specimens 1 and 2, respectively, A is the amplitude of the wave, f is the frequency, x is the distance travelled by the wave, GF is a frequency independent geometrical factor which includes spreading, reflections etc., V is the wave velocity and Q is the quality factor which can be correlated to the damping ratio ξ using Eq. (2).

$$\frac{1}{Q} = 2\xi \tag{2}$$

However, the spectral ratio method has various shortcomings. The first configuration of the spectral ratio method carries the implicit assumption that the frequency spectra of the reference specimen's signal and the tested specimen's signal have the same frequency range. Difference in specimens' stiffness will lead to mismatch in the frequency spectra leading to a large scatter in the damping ratio when using SRM [9]. The second configuration suffers from practical limitation as it is almost impossible to obtain identical soil specimens of different heights unless they are reconstituted specimens.

Another widely popular method used to determine damping ratio is the Logarithmic Decrement Method (LDM). The LDM is utilised in resonant column tests to determine the damping ratio of soil specimens [10]. However, the LDM only works well in the resonant column test where the entire soil specimen was subjected to steady state vibration at its resonant frequency and the excitation was cut off to allow the free vibration decay curve to be obtained [10]. In the bender element test where only a small perturbation is introduced into the soil specimen, the transient nature of the propagating wave is easily affected by

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https://doi.org/10.1016/j.soildyn.2018.04.016
Received 29 March 2017; Received in revised form 6 April 2018; Accepted 11 April 2018

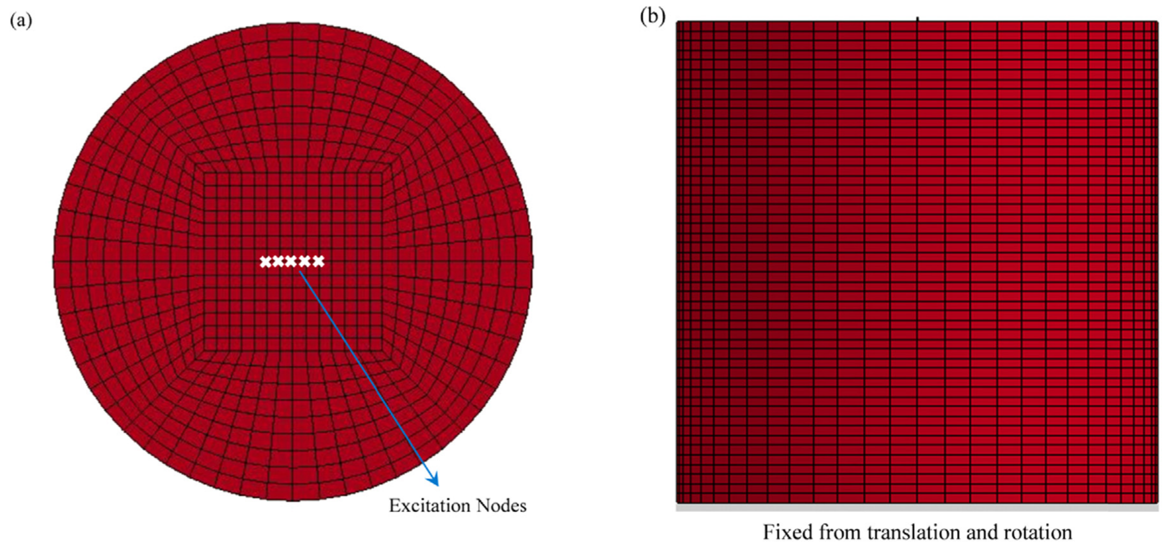

Fig. 1. (a) Top and (b) front view of finite element model.

Table 1
Input parameters for viscoelastic model (MAT_006).

Density, ρ	2096 kg/m^3
Bulk modulus, K	6274 MPa
Short term shear modulus, G_o	168 MPa
Long term shear modulus, G_∞	0 MPa
Decay constant, β	See Eq. (9)

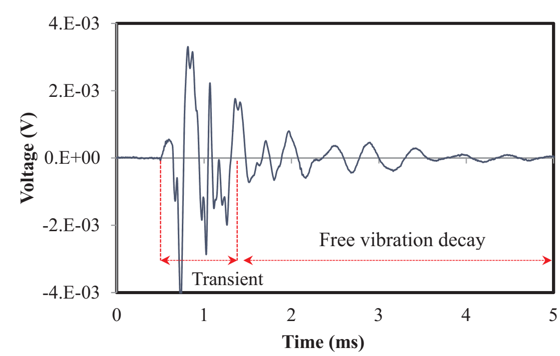

Fig. 2. Typical experimental S-wave signal from the bender element test.

interference from reflected waves. This interference give rises to irregularities which cannot be removed via digital signal processing and would thus affect the application of the LDM. Moreover, the number of decay cycles is usually insufficient to apply LDM reliably [11]. In view of the limitations of bender element test, the HTM is able to provide more reliable results than LDM as demonstrated in the finite element simulations and experimental results presented.

The Hilbert transform method (HTM) is conceptually similar to the LDM and was first used by Agneni and Balis-Crema [12] to derive damping ratios of composite materials using free vibration decay data. The Hilbert transform is an operator which convolutes a signal by $1/\pi x$. In other words, it is a filter which transforms the signal by shifting their phases by $\pm\pi/2$ while maintaining the magnitudes of their respective spectral components. In most cases, determination of the dynamic response of engineering structures involves measuring vibration responses subjected to random wind and other background vibrations. These engineering structures respond in different modes of vibration. Determination of damping ratio thus requires the use of the ‘empirical mode decomposition’ method [13] to decompose complicated signals into their respective modal components to yield well-behaved Hilbert transforms [13–19]. However, in bender element test where a single sinusoidal pulse is introduced and the receiver bender element records the free vibration decay in a known mode (flexural for shear wave), such decomposition of the signal is not required.

In HTM, if x(t) is the time domain signal (Eq. 3a) and $x^H(t)$ is the Hilbert transform of the time domain signal (Eq. 3b), the combination will give the analytic signal $x_a(t)$ (Eq. 3c) [14,19].

$$x(t) = Ae^{-\xi\omega_n t}\sin(\omega_n t\sqrt{1-\xi^2}) \tag{3a}$$

$$x^H(t) = Ae^{-\xi\omega_n t}\cos(\omega_n t\sqrt{1-\xi^2}) \tag{3b}$$

$$x_a(t) = x(t) - ix^H(t) \tag{3c}$$

where A is the amplitude of the signal, ξ is the damping ratio, ω_n is the natural frequency in radians, t is the time in seconds and i is the imaginary number. The magnitude of the analytic signal gives the time signal $x_a(t)$ and is shown in Eq. (4).

$$|x_a(t)| = Ae^{-\xi\omega_n t} \tag{4}$$

From Eq. (4), the damping ratio ξ can be separated as shown in Eq. (5).

$$\ln(|x_a(t)|) = \ln(A) - (\xi\omega_n)t \tag{5}$$

Eq. (5) shows that the gradient from the plot of $\ln(|x_a(t)|)$ with t used to derive the damping ratio has to be negative. The damping ratio can be derived from the gradient m (= $\xi\omega_n$) as shown in Eq. (6). When considering the time window to obtain the gradient, it is important to avoid both ends of the analytic signal which can be distorted by the Hilbert transform [20,21].

$$\xi = \frac{m}{\omega_n} = \frac{m}{2\pi f_n} \tag{6}$$

where m is the gradient and f_n is the natural frequency of the signal.

Eq. (6) shows that in order to obtain the damping ratio, the natural frequency of the signal will be necessary. The situation is not straightforward as the signal detected in bender element test is a broad frequency band signal and the natural frequency is affected by the soil properties [22]. Lee and Santamarina [23] showed that the first resonant frequency (f_r) of an equivalent bender element–soil system can be obtained as: