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Sensitivity analysis and calibration of phenomenological models for seismic analyses

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ABSTRACT

Phenomenological models used in seismic structural analyses are often based on parameters without explicit physical meaning, which must be calibrated by fitting experimental responses. Parameter calibration, as an inverse problem, may suffer from ill-posedness, and thus the results are always to be critically examined before accepting them. In this paper, a comprehensive methodology, comprising repeated optimisation runs, local and global sensitivity analysis and simplified uncertainty analysis is described with the aim of providing some guidelines to assess the calibration results. As exemplary case study, the calibration of a phenomenological model for steel members by means of a series of experimental tests is presented. The experimental response of nominally identical beams tested under monotonic, cyclic and pseudo-dynamic loading were used in the procedure. The main findings of the work indicate that the optimisation process based on Genetic Algorithms is able to find optimal solutions in terms of fidelity to the experimental tests: However, being the problem ill-posed, the same level of fitting may be attained by solutions characterised by different model parameters. Local and global sensitivity analyses may help assess the identifiability of the parameters, while a-posteriori uncertainty analysis provides an estimation of the uncertainty in the prediction. It is shown that increasing the number of calibration tests may reduce the ill-conditioning of the problem, and thus a multi-objective approach is strongly recommended. Finally, a novel procedure recently developed based on tolerance-based Pareto dominance is shown to give similar results to those provided by computationally expensive sensitivity analyses at the computational cost of a single calibration analysis.

1. Introduction

Engineering predictions of structural seismic response are based on mathematical representation of the physical behaviour of mechanical systems. To be accurate, a mathematical model should represent the structural behaviour either in the elastic or nonlinear branch, as well as in monotonic or cyclic conditions. While good approximations could in principle be obtained by using detailed Finite Element discretisation of the differential equations governing the problem [1–4], this approach is not feasible when large-scale structures are to be studied, because of the computational cost needed and the impracticability of the definition of the single phenomena occurring in the components. For these reasons, it is generally preferred to apply a *phenomenological* approach, in which simplified numerical models are used to represent a behavioural response regardless of the underlying physics. Consider for instance flexural strength degradation occurring in a steel member under cyclic loading, which is well-known to be due to local buckling of compressed parts in the cross-section [5]: a mechanics-based approach would consider modelling the initial imperfection in the component and its effect on the local stress distribution and the stability of equilibrium. On the contrary, a phenomenological approach recognises the phenomenon at the larger scale (strength degradation) and tries to define a mathematical model relating this to some global parameters, as dissipated plastic energy or ductility. This change of perspective has two important consequences. Firstly, the different scale of description allows the analyst to reduce the computational burden of the analysis yet accounting for all important nonlinear effects on the structure. Secondly, the model becomes independent from the physics of the problem, and can represent responses at very different scale (from stress-strain relationship of a material to base shear-top displacement of a building) without conceptual differences.

An example of phenomenological approach in structural analysis is the concentrated-plasticity strategy, according to which nonlinear behaviour of members (typically beams in frame structures) is supposed to

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manifest in limited regions known a priori [6,7], modelled as zero- or finite-length nonlinear elements. Different formulations for the nonlinear elements are provided by the models by Takeda [8], Bouc and Wen [9,10], Ramberg and Osgood [11], Richard and Abbott [12], Dowell, Seible and Wilson [13], Sivaselvan and Reinhorn [14], Ibarra, Medina and Krawinkler [15].

As most of these models rely on some parameters whose physical meaning is not immediately recognisable, model calibration is usually based on curve fitting of experimental data coming from purposely designed tests [16]. Hence, the calibration problem becomes an optimisation problem in which a function of the mismatch between experimental data and numerical counterparts computed from a model with a given choice of parameters is minimised. The main issue in this approach is that due to various model and data errors and diverse sensitivity of the model parameters, several parameter combinations may give similar levels of fidelity to the response taken into consideration, yet providing completely different quality in the prediction of the response under other loading conditions. This issue was raised for instance in [17] with regards to a hysteretic model for steel members calibrated against data coming from cyclic tests following ordinary protocols. It was also suggested therein that the addition of further information, i.e. tests to fit, in the framework of multi-objective optimisation could be beneficial for increasing the robustness of the results. As those results show that calibration of phenomenological models based on curve fitting is far from being a routine task, it is recommended that a critical assessment of the results should answer the following questions:

1. Is the optimisation procedure robust?
2. What is the level of confidence that can be assigned to the calibrated parameters?
3. How does the uncertainty of the calibrated parameters relate to the response prediction?

The answer to such fundamental questions involves careful consideration of the optimisation algorithm, sensitivity analysis (SA) methodologies and relates to the general area of calibration and validation [18–21]. In [22] a framework for quantifying the uncertainty in the calibration of mechanical models and how it propagates in the prediction is proposed in the context of Bayesian updating. In [23] Ma et al. show how the use of SA may be useful in determining the importance (and thus the identifiability) of the parameters of the Bouc-Wen model. The use of SA in the determining the relative importance of input parameters for a model is now widespread in environmental modelling [24], statistical sciences [25,26] and chemical modelling [27], while in structural engineering it appears to be slightly less common.

In this work, following the calibration methodology proposed in [17], a comprehensive assessment procedure comprising repeated optimisation runs, local and global sensitivity analysis and simplified uncertainty analysis will be described and applied to a case study involving the calibration of a structural phenomenological model for beam plastic zones. Even though the numerical results are strictly valid for the application considered only, the main motivation for this work is suggesting general guidelines for performing accurate calibration analyses and promoting good practice in the field of structural model calibration. Moreover, a novel method for calibration accounting for tolerance in the objective satisfaction is finally shown to provide results similar to those given by SA at a cost of a single optimisation run.

2. Overview of the calibration procedure and assessment strategy

2.1. The calibration problem

Calibration (or parameter identification) of a numerical model means finding the set of parameters $\tilde{\boldsymbol{p}}$ such that the computed response given by the simulation of a test $\boldsymbol{y_c}(\boldsymbol{p})$ is as close as possible to the experimental response $\boldsymbol{y_{exp}}$. This implies solving the optimisation problem:

$$\tilde{\boldsymbol{p}} = \underset{\boldsymbol{p}\in\boldsymbol{P}}{\operatorname{argmin}}\, \omega\,(\boldsymbol{y_{exp}}, \boldsymbol{y_c}(\boldsymbol{p})) \tag{1}$$

where $\omega(\boldsymbol{y_{exp}}, \boldsymbol{y_c}(\boldsymbol{p})) = \omega(\boldsymbol{p})$ is a suitable discrepancy function measuring the inconsistency between the experimental and computed quantities, and $\boldsymbol{P}$ the set of all possible parameter combinations. One of the simplest and most widespread formulation for the discrepancy function, adopted in this work, is:

$$\omega(\boldsymbol{p}) = \frac{1}{\omega_{\text{ref}}}\|\boldsymbol{y_{exp}} - \boldsymbol{y_c}(\boldsymbol{p})\| \tag{2}$$

where $\|\bullet\|$ represents the Euclidean norm of a vector and $\omega_{\text{ref}} = \|\boldsymbol{y_{exp}}\|$ is a scaling factor needed to make ω non-dimensional.

When N_T calibration tests are performed, the optimisation problem (1) is replaced by:

$$\tilde{\boldsymbol{p}} = \underset{\boldsymbol{p}\in\boldsymbol{P}}{\operatorname{argmin}}\left\{\omega_1(\boldsymbol{p}), \ldots, \omega_{N_T}(\boldsymbol{p})\right\} \tag{3}$$

where $\omega_i(\boldsymbol{p})$ represents the discrepancy value of the i-th test.

In the context of multi-objective optimisation, the concept of Pareto optimality replaces the usual notion of optimality [28]. In a minimisation problem, a solution $\boldsymbol{p_1}$ is said to dominate a solution $\boldsymbol{p_2}$ if and only if:

$$\begin{aligned} \omega_i(\boldsymbol{p_1}) &\leq \omega_i(\boldsymbol{p_2}) \quad \forall\, i = 1, \ldots, N_T \\ \omega_i(\boldsymbol{p_1}) &< \omega_i(\boldsymbol{p_2}) \quad \exists\, i = 1, \ldots, N_T \end{aligned} \tag{4}$$

A solution is referred to as Pareto optimal if it is not dominated by any other solution. The ensemble of all Pareto optimal solutions in the objective space is said Pareto Front $\boldsymbol{PF}$, and it represents the general solution of (3). As a typical configuration of the Pareto Front for minimisation problems (in two dimensions) is L-shaped (Fig. 1), if all objectives are correctly scaled (as, for instance, by means of formulation (2)) a unique compromise solution $\boldsymbol{p_{compr}}$ may be extracted from the Pareto Front, according to the rule:

$$\boldsymbol{p_{compr}} = \underset{\boldsymbol{p}\in\boldsymbol{PF}}{\arg\min}\, \|\boldsymbol{\omega}(\boldsymbol{p}) - \boldsymbol{\omega_{utopia}}\| \tag{5}$$

where $\boldsymbol{\omega}(\boldsymbol{p}) = [\omega_1(\boldsymbol{p}), \ldots, \omega_{N_T}(\boldsymbol{p})]^T$ and $\boldsymbol{\omega_{utopia}}$ is the vector of the minimum discrepancies. This rule corresponds to selecting from $\boldsymbol{PF}$ the nearest solution to the utopia point where all objectives have minimum value (Fig. 1).

The optimisation problem (1) or (3) may be solved by using different algorithms (gradient-based, meta-heuristics). Genetic Algorithms

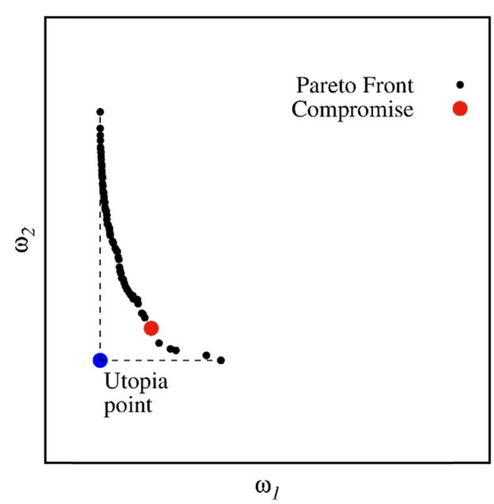

Fig. 1. Typical Pareto Front in two-objective minimisation problems and selection of the compromise solution.