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Love wave frequency in an orthotropic crust over a double-layered anisotropic mantle

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ABSTRACT

In the present paper, investigation has been made to study the propagation character of Love wave in an orthotropic substratum being layered doubly over anisotropic porous mantle. Separate expressions for displacement components have been derived for each of the three layer and suitable boundary conditions have been imposed to derive the dispersion equation of the wave in a closed form in terms of Whittaker function. It has been found that inhomogeneity parameter associated with the rigidity and the density of the layered medium has a special bearing on the Love wave propagation. Orthotropic properties and initial stresses of the media also affects the velocity of the wave to a great extent has been shown graphically.

1. Introduction

It is desirable to study the Love wave propagation in an anisotropic media because the propagation of elastic waves in anisotropic and orthotropic media is fundamentally different from their propagation in isotropic media. Because the earth's crust and half space (mantle) are inhomogeneous, it is very interesting to know the propagation pattern of Love wave in a heterogeneous medium as is studied sufficiently by Shearer [1]. It has been observed that the propagation of Love wave is influenced by the elastic properties of the medium through which it travels to a great extent. The earth's crust contains some hard and soft rocks or materials that may exhibit orthotropic property and porosity. In Orthotropic material, the mechanical or thermal properties are quite unique and independent along the three mutually perpendicular Cartesian axes. These facts motivate us to investigate further on Love wave propagation. Destrade [2] studied surface waves in orthotropic incompressible materials whereas Kumar and Rajeev [3] analyzed the wave motion at the boundary surface of orthotropic thermoelastic material with voids. Recently, Ahmed and Dahab [4] demonstrated Love wave propagation in an orthotropic granular layer under initial stress while a clear picture have been explained by Kumar and Choudhary [5] on responses of orthotropic micropolar elastic medium due to various sources. Various problems in the field of seismology may be solved by treating the earth as a layered medium with certain thickness and mechanical properties. It was Pujol [6] and Chapman [7] who studied in detail about the elastic wave propagation and its generation in seismology. Singh [8] demonstrated the behaviors of Love wave in layered medium where surface has irregular shapes while an interesting study made by Ke et al. [9] on propagation of Love waves in a heterogeneous fluid saturated porous layered mantle with linearly varying properties. In the theoretical study of elastic and seismic waves, mathematical expression provides a bridge between modeling results and field application. These studies are consisting of finding a solution of system of partial differential equations under a certain imposed initial and boundary conditions. The propagation of seismic waves in the interior of the earth is governed exactly by mathematical laws similar to the laws of light waves in optics.

Love wave and its dispersion in an orthotropic layer lying over a half space have its significant role in earthquake engineering and seismology on account of the occurrence of the inhomogeneity in the earth's crust and mantle. Dai and Kunag [10] studied Love waves in double porosity media and Ghorai [11] investigated Love waves in a fluid[HYPHEN]saturated porous layer under a rigid boundary lying over an elastic half space under gravity while Son and Kang [12] explained the propagation of shear waves in a poro-elastic layer constrained between two elastic layers. In general, the holes and pores contain the layer of hydrocarbon deposited in the form of gas and oil. These deposits of gases and oils are found in the form of sandstone or limestone which is like hard sponge full of holes though not compressible. These holes or pores in turn may contain fluids such as water or oil along with gas which may further be saturated with rock called as porous layer. Hence, the influence of porosity on the propagation of seismic waves drew the attention of several authors. Basing the theory explained by Biot [13], the problems of wave propagation through

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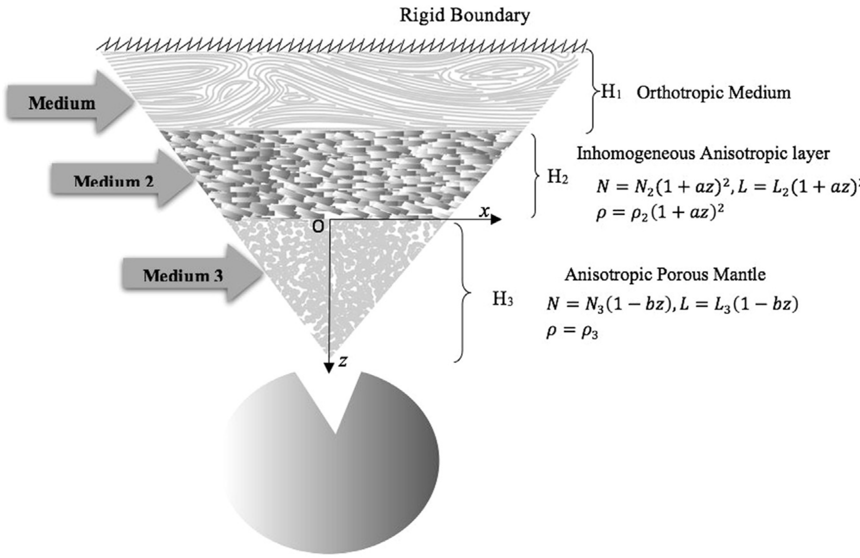

Fig. 1. Geometry of the problem – spherical earth model with three layered stratum.

porous media have been discussed in detail by several authors, viz. Shams [14], Zhang et al. [15], Kaptsov and Kuznetsov [16], Qian et al. [17], Sharma and Gogna [18], Gardner [19] and others. Investigation has been made on Finite-amplitude Love waves in a pre-stressed compressible elastic half space with a double surface layer by Kayestha et al. [20] whereas Midya [21] studied Love-type surface waves in homogeneous micropolar elastic media. References can also be made to Han et al. [22], Chattaraj et al. [23], Singh et al. [24], Vishwakarma et al. [25] and Gupta et al. [26] for their interesting work over seismic wave propagation in various geo-media. The surface waves-based seismic exploration of soil and ground water have been explained by Serdyukov et al. [27] while surface wave radiation patterns that involve amplitude and initial phase were determined for several larger underground nuclear explosion and small-magnitude earthquakes by Brune and Pomeroy [28]. Analysis of dispersion and attenuation of surface waves in poroelastic media in the exploration-seismic frequency band have been studied by Zhang et al. [29].

In the present study, the crustal part of the earth is taken as orthotropic medium, followed by an inhomogeneous anisotropic layer supported over anisotropic porous mantle. Quadratic variations have been taken in the directional rigidities and density of the sandwiched layer i.e. $N = N_2(1 + az)^2$, $L = L_2(1 + az)^2$ and $\rho = \rho_2(1 + az)^2$, where a is a constant called inhomogeneity parameter having dimension that of inverse of length. Based on the theories as given by Holness [30], it can be claimed that fluid saturated anisotropic porous medium may affect the propagation behavior of surface waves. However, experimental studies demonstrate that basaltic melt occupies a pore structure in mantle materials as the rock approaches or attains textural equilibrium. In crystallizing plutons the geometry of melt-filled pores during the last stages of crystallization can be inferred from the disposition of the last crystalizing phases. In pelitic rocks that were once partially molten, textural studies have shown that the melt-filled porosity can be pseudomorphed by minerals such as quartz or alkali feldspar. The mantle being porous in nature is assumed to have kinetic isotropy, but elastic anisotropy of Weiskopf type, where the viscosity of water is neglected. Hence, Linear variations have been taken in the half space (mantle) i.e., $N = N_3(1 - bz)$, $L = L_3(1 - bz)$, $\rho = \rho_3$ where b is an inhomogeneity parameter with same dimension as that of a. When the values of b > 0, the directional rigidities will decrease with the depth while for b < 0, it will increase with the depth. However, Bilek and Lay [31] have discussed in detail the rigidity variations with depth along interplate megathrust faults in subduction zones, and based on the theory being presented, it is reasonable to say that in some part of the earth, the rigidity decreases with depth. Therefore, the present study has been made to come up with the effect of such medium on the phase velocity of Love wave propagation. Hence, as far as the real earth model is concerned, the geometry of the problem is not very common but yet it exists as per the theories available. Keeping this in mind, the present study has been made to study how does Love wave will behave under the influence of multi-layered geometry with anisotropic medium being at the bottom where rigidity decreases and increases as per the positive and negative values of inhomogeneity parameter 'b' associated with it. Suitable boundary condition under the assumption of rigid boundary plane have been considered and imposed on the displacement of the wave which have been found for individual layers. The frequency equation (dispersion equation) has been derived in closed form along with the classical equation of Love wave given by Love [32]. The numerical values of the phase velocity have been calculated using the values of material constants given by Biot [33] from experiments and the effect inhomogeneity parameter associated with rigidity and density is discussed and demonstrated using graphs.

2. Formulation of the problem

The earth is highly inhomogeneous in nature, its crustal layers and mantle is composed of variety of medium. Keeping in mind the real scenario of the globe, the problem is constructed taking into consideration three medium viz. Orthotropic layer (H_1 thickness), Inhomogeneous anisotropic layer (H_2 thickness) and Anisotropic porous medium, where the propagation of Love wave in the orthotropic crustal layer has to be investigated under the influence of rigid boundary at the top in the presence of various inhomogeneity associated in the density and rigidity of the media as shown in the Fig. 1.

3. Displacement of wave in orthotropic layer (medium 1)

The upper crustal layer taken in the problem is orthotropic in nature, the layer being under the influence of initial stress P_1 along x direction as shown in Fig. 1, the equation of motion in the absence of body forces are given by Biot [33]

$$\begin{cases} \frac{\partial\sigma_{11}}{\partial x} + \frac{\partial\sigma_{12}}{\partial y} + \frac{\partial\sigma_{13}}{\partial z} - P_1\left(\frac{\partial w_z}{\partial y} - \frac{\partial w_y}{\partial z}\right) = \rho_1\frac{\partial^2 u_1}{\partial t^2} \\ \frac{\partial\sigma_{21}}{\partial x} + \frac{\partial\sigma_{22}}{\partial y} + \frac{\partial\sigma_{23}}{\partial z} - P_1\left(\frac{\partial w_z}{\partial x}\right) = \rho_1\frac{\partial^2 v_1}{\partial t^2} \\ \frac{\partial\sigma_{31}}{\partial x} + \frac{\partial\sigma_{32}}{\partial y} + \frac{\partial\sigma_{33}}{\partial z} - P_1\left(\frac{\partial w_y}{\partial x}\right) = \rho_1\frac{\partial^2 w_1}{\partial t^2} \end{cases} \tag{1}$$