Soil Dynamics and Earthquake Engineering 108 (2018) 46–57

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Finite-difference modeling and characteristics analysis of Rayleigh waves in anisotropic-viscoelastic media

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ARTICLE INFO

Keywords:
Rayleigh waves
Finite-difference
Anisotropic
Viscoelastic
Attenuation
Dispersion

ABSTRACT

Effects of anisotropy and viscoelasticity of Earth media on Rayleigh waves propagation are studied by numerical simulation based on a finite-difference (FD) scheme, which calculates the spatial derivatives via a 12th-order staggered grid difference operator and the time derivatives through the fourth-order Runge-Kutta method. In the homogeneous half-space, the accuracy of FD algorithm is tested against the isotropic-elastic (IE) analytical solution; the calculation method of the theoretical phase velocities of the anisotropic-viscoelastic (AV) Rayleigh waves is developed to verify the correctness of modeling results; the characteristics of Rayleigh-wave are analyzed by comparisons among IE, anisotropic-elastic (AE) and AV modeling results in terms of the wave field snapshots, the synthetic seismograms, and the dispersive images, respectively. Then, the two-layer models and the four-layer models are utilized to further analyze the characteristics of Rayleigh waves in the AV layered media. Results show the substantial differences among these three kinds of media. Anisotropy of medium leads to the significant changes of Rayleigh-wave in amplitude, waveform and phase velocity due to anisotropy of velocity, but does not cause the phase velocity dispersion of Rayleigh-wave. Viscoelasticity of media arouses the amplitude attenuation and phase velocity dispersion of Rayleigh-wave. This is the first report for FD modeling and characteristics analysis of Rayleigh-wave in the AV media, which will provide a valuable reference for near-surface geophysical investigation.

1. Introduction

Rayleigh-wave, which is the result of interfering P- and SV-waves at the free surface and travels along the free surface, is highly regarded in seismology fields, especially in near-surface geophysical exploration [1–3]. Rayleigh-wave analysis aimed at estimating the subsurface shear-wave velocity profile can be performed according to the different methods. The classical Multichannel Analysis of Surface Waves (MASW) method [4–6], which is based on the analysis of the vertical component of Rayleigh waves, consists of the acquisition of field data [7–9], extraction of modal dispersion curves [10–12], and inversion of phase velocities [13–16], and has become one of the main seismic methods for applications of geotechnical and environmental engineering. However, in some cases, due to complexity of the modal dispersion energies (e.g., mode-kissing [17]), the MASW method in terms of modes can generate the incorrect or inaccurate shear-wave velocity because of mode misidentification. Therefore, some of the further methods, such as the technologies of Full Waveform Inversion (FWI) [18,19] and Full Velocity Spectrum (FVS) inversion [3], the methods of joint analysis (e.g., the different components of Rayleigh waves [20]; Rayleigh waves and Love waves [21], or refraction events [22], or reflection events [15], or Horizontal-to-Vertical Spectral Ratio (HVSR) [23]), the method of Rayleigh-wave Particle Motion (RPM) analysis [24], and the methods of passive surface waves (e.g., Multi-channel Analysis of Passive Surface Waves (MAPS) [25], Miniature Array Analysis of Microtremor (MAAM) [26]), were proposed or developed to solve this sort of possible issues, or to improve the estimation accuracy of shear-wave velocity from the different standpoints. In order to obtain more real results, Rayleigh-wave analysis methods should take into account the natures of the Earth's medium.

The Earth has been widely recognized as the anisotropic-viscoelastic (AV) medium with the significant effects on seismic waves propagation [27–30]. Effects of viscoelasticity are mainly in two aspects [31,32]: (1) amplitude attenuation—amplitudes decay with propagation distance due to energy loss and (2) velocity dispersion—velocities vary with frequency due to the relaxation of stress and strain. The rheological

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https://doi.org/10.1016/j.soildyn.2018.02.004
Received 7 August 2017; Received in revised form 2 February 2018; Accepted 3 February 2018

behavior of Earth media can be modeled using viscoelastic models, such as the generalized Zener body (GZB) [33–35] and the generalized Maxwell body (GMB) [36–38], which combines behaviors of elastic solids and viscous fluids [39]. Anisotropy of Earth media makes the seismic waves propagation velocities different in different direction, causes the direction of energy rays inconsistent with the wavenumber direction, and introduces a typical phenomenon named as shear-wave splitting [40–43]. Transversely isotropic (TI) solid has been widely used for studying the anisotropy of Earth media, and it can be further simplified to transversely isotropic solid with a vertical symmetry axis (Vertical Transversely Isotropic, VTI) [44,45].

Effects of anisotropy and viscoelasticity on Rayleigh waves are actually more complicated than on body waves because of the rigorous generation conditions of Rayleigh-wave, and there are a large number of researchers attracted to study Rayleigh waves in the anisotropic or viscoelastic media from the different aspects. Several researchers investigated theoretically the characteristics of viscoelastic Rayleigh waves from the standpoint of energy and pointed out that phase velocities and attenuation factors give a good approximation to the dispersive and dissipation of Rayleigh-wave [46,47]. Several researchers analyzed the propagation and dispersion characteristic of viscoelastic Rayleigh waves by the wave fields numerical modeling, and modeling results present the substantial differences compared with pure elastic cases [48,49]. Some researchers estimated the near-surface quality factors by constrained inversion of attenuation coefficients extracted from amplitude information of viscoelastic Rayleigh waves [50,51]. Some researchers studied the propagation and dispersion characteristics of Rayleigh waves from the standpoint of phase velocity of Rayleigh waves calculated from the dispersion equation in layered anisotropic-elastic (AE) medium [52–55]. To better understand Rayleigh-wave behaviors in Earth media, it is necessary to study the characteristics of Rayleigh waves via numerical modeling in a more realistic medium described by an AV rheology, however, which has not been reported by researchers at present.

The purpose of this work is to study the characteristics of Rayleigh waves in VTI-viscoelastic media through numerical modeling. In order to obtain modeling results with high accuracy, we utilized a finite-difference (FD) modeling scheme, which calculates the spatial derivatives by a 12th-order staggered grid difference operator [56] and the time derivatives via the fourth-order Runge-Kutta method [48], to solve the first-order P-SV velocity-stress VTI-viscoelastic wave equations. The free-surface boundary is treated using the Stress Image Method (SIM) [57,58]. The Multiaxial Perfectly Matched Layer (M-PML) [59,60], which is adopted to treat the artificial boundaries, is first used for the FD scheme of the VTI-viscoelastic wave equations. We also extended the calculating method of the theoretical phase velocities of Rayleigh waves in the homogeneous half-space from the AE media to the AV media, which are used for verifying the correctness of modeling results.

In this paper, we simulated the AV Rayleigh-wave in the half-space models and the layered models by FD method. First, we introduced the methodology, which involves the wave equations, the boundary treatments, and the modeling scheme. Then, in the homogeneous half-space models, we tested the modeling program against the elastic analytical solution in time-space (*t-x*) domain, verified the correctness of modeling results via comparing with the theoretical phase velocities of Rayleigh-wave in frequency-velocity (*f-v*) domain, and analyzed the characteristics of Rayleigh waves by the comparisons among IE, AE and AV modeling results in terms of the wave field snapshots, the synthetic seismograms, and the dispersive images, respectively. Finally, in the layered models including the two-layer models and four-layer models, we further investigated the performance of our codes in modeling Rayleigh-wave, and analyzed the characteristics of Rayleigh waves in the AV layered media.

2. Modeling method

2.1. The wave equations

The first-order P-SV velocity-stress wave equations of the 2-D VTI-viscoelastic media in time domain can be derived from the momentum conservation equation and the particular constitutive relationship, and they can be formed with the following equations.

The linearized equations of momentum conservation are [34,61]:

$$\dot{v}_x = \frac{1}{\rho}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z}\right) + f_x, \tag{1a}$$

$$\dot{v}_z = \frac{1}{\rho}\left(\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z}\right) + f_z, \tag{1b}$$

where v_x and v_z are the particle velocities; σ_{xx}, σ_{xz} and σ_{zz} are the stress components; ρ denotes the mass density; f_x and f_z are the body forces. The dot above a variable indicates time differentiation.

The constitutive equations are [61]

$$\dot{\sigma}_{xx} = \hat{c}_{11}\frac{\partial v_x}{\partial x} + \hat{c}_{13}\frac{\partial v_z}{\partial z} + (D - c_{55})\sum_{l=1}^{L_1} \dot{e}_{1l} + c_{55}\sum_{l=1}^{L_2} \dot{e}_{2l}, \tag{2a}$$

$$\dot{\sigma}_{zz} = \hat{c}_{13}\frac{\partial v_x}{\partial x} + \hat{c}_{33}\frac{\partial v_z}{\partial z} + (D - c_{55})\sum_{l=1}^{L_1} \dot{e}_{1l} - c_{55}\sum_{l=1}^{L_2} \dot{e}_{2l}, \tag{2b}$$

$$\dot{\sigma}_{xz} = \hat{c}_{55}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) + c_{55}\sum_{l=1}^{L_2} \dot{e}_{3l}, \tag{2c}$$

where

$$\hat{c}_{11} = c_{11} - D + (D - c_{55})M_{u1} + c_{55}M_{u2}, \tag{3a}$$

$$\hat{c}_{13} = c_{13} + 2c_{55} - D + (D - c_{55})M_{u1} - c_{55}M_{u2}, \tag{3b}$$

$$\hat{c}_{33} = c_{33} - D + (D - c_{55})M_{u1} + c_{55}M_{u2}, \tag{3c}$$

$$\hat{c}_{55} = c_{55}M_{u2}, \tag{3d}$$

are the high-frequency limit anisotropic elasticity constants, with

$$\begin{bmatrix} c_{11} & c_{13} & 0 \\ & c_{33} & 0 \\ & & c_{55} \end{bmatrix} \tag{4}$$

the symmetric 2-D low-frequency limit anisotropic elasticity matrix, and $D = (c_{11} + c_{33})/2$. They can be expressed by Thomsen coefficients as [40]

$$c_{11} = \rho(1 + 2\varepsilon)v_{P0}^2,\ c_{33} = \rho v_{P0}^2,\ c_{55} = \rho v_{S0}^2, \tag{5a,b,c}$$

$$c_{13} = \rho v_{P0}^2\sqrt{f(f + 2\delta)} - \rho v_{S0}^2,\ f = 1 - v_{S0}^2/v_{P0}^2, \tag{5d}$$

where v_{P0} and v_{S0} are respectively the vertical (radial) elastic P- and S-wave velocities. The dimensionless parameters ε and δ are anisotropy coefficients in which ε indicates the anisotropy of P-wave velocities, and δ reflects the difference of P- and S-wave. The unrelaxed moduli M_{u1} and M_{u2} are defined by equation [34]:

$$M_{u\nu} = 1 - \frac{1}{L_\nu}\sum_{l=1}^{L_\nu}\left(1 - \frac{\tau_{\varepsilon l}^{(\nu)}}{\tau_{\sigma l}^{(\nu)}}\right),\ \nu = 1, 2, \tag{6}$$

where $\tau_{\varepsilon l}^{(\nu)}$ and $\tau_{\sigma l}^{(\nu)}$ are material relaxation times. $M_{u\nu}$ are relaxation functions evaluated at $t = 0$, with $\nu = 1$, the quasi-dilatational mode, and $\nu = 2$, the quasi-shear mode. In the constitutive equations, the quantities e_{1l} are memory variables related to the L_1 mechanisms which describe the viscoelastic characteristics of the quasi-dilatational wave, and e_{2l} and e_{3l} are the memory variables related to the L_2 mechanisms for the quasi-shear wave.

The memory variable first-order equations in time are [61]