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Exact solution to scattering of SH waves by an elliptic-arc canyon in the corner of an elastic quarter space

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ABSTRACT

Herein, we investigate the dynamic anti-plane problem of an elliptic-arc canyon in the corner of an elastic quarter space. The method of wavefunction expansions and the method of images are applied to derive an exact analytical solution for plane-wave incidence. Values of field quantities are evaluated via closed-form expressions involving Mathieu functions. Results are given as benchmarks for numerical studies.

1. INTRODUCTION

Issues concerning the scattering effect of seismic waves remain a topic of current interest (e.g. [2]). However, exact analytical solutions are available only for a limited number of geometries (e.g. [6]). Regarding the problem involving scattering of plane SH waves by an elliptic-arc canyon in the corner of an elastic quarter space (e.g. [3]), no reliable results are given in existing literature to date. Consequently, in this short note we aim to address this concern straight away. Based on the method of separation of variables in elliptic coordinates, the authors derive an exact analytical solution.

2. FORMULATIONS

The assumed problem geometry is depicted in Fig. 1(a), where an elliptic-arc canyon (with width a, depth b, and half-focal distance c) in the corner of an homogeneous, isotropic, linearly elastic, quarter space is considered. The soil medium has the shear modulus μ and the shear-wave velocity c_s. An infinite train of plane SH waves (with an angular frequency ω) is incident upon the canyon at an angle α with respect to x-axis. The vertex of the quarter space (coinciding with the center of the canyon) is taken as the origin of the Cartesian coordinate system (x, y) and the elliptic coordinate system (ξ, η). Throughout this section, the time-harmonic factor $\exp(-i\omega t)$ is understood.

The steady-state out-of-plane motions in the region concerned have to obey the reduced wave equation, namely

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} + a^2 k^2 (\cosh^2 \xi - \cos^2 \eta) u = 0 \tag{1}$$

where $k = \omega / c_s$ is the shear wavenumber.

The stress-free boundary conditions are imposed on the top edge of the quarter space,

$$\frac{\partial u(\xi, \eta)}{\partial \eta} = 0, \;\; \eta = -\pi \,, \tag{2}$$

on the lateral edge of the quarter space,

$$\frac{\partial u(\xi, \eta)}{\partial \eta} = 0, \;\; \eta = -\frac{\pi}{2} \,, \tag{3}$$

and on the surface of the canyon,

$$\frac{\partial u(\xi, \eta)}{\partial \xi} = 0, \;\; \xi = \xi_0 \,. \tag{4}$$

In the following subsections, two methods are applied to construct the solution to the present boundary-value problem. Note that the angle of incidence defined for deep canyons is different from that for shallow canyons (cf. Figs. 2a and 2d).

2.1. Method 1

The incident waves u^I may be written as

$$u^I = \exp\left[ik(x\cos\alpha + y\sin\alpha)\right] . \tag{5}$$

For the quarter-plane medium without any anomalies, the free-field displacement u^F may be expressed as a sum of the incident waves u^I and their reflected waves u^{R1}, u^{R2}, and u^{R3} (see Fig. 1b). Thus,

$$u^F = u^I + u^{R1} + u^{R2} + u^{R3} \tag{6}$$

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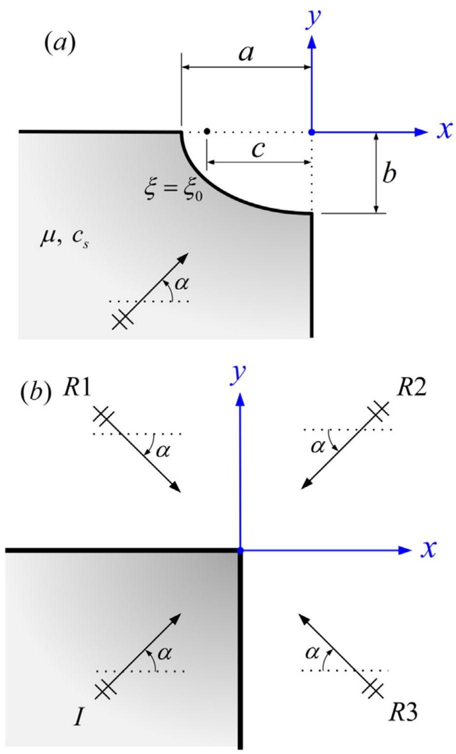

Fig. 1. (a) Geometric layout of the problem. (b) Principle of superposition applied to construct the free-field displacements in a quarter space.

where

$$u^{R1} = \exp[ik(x\cos\alpha - y\sin\alpha)]\,, \tag{7}$$

$$u^{R2} = \exp[-ik(x\cos\alpha + y\sin\alpha)]\,, \tag{8}$$

$$u^{R3} = \exp[-ik(x\cos\alpha - y\sin\alpha)]\,. \tag{9}$$

From Eq. (6), we have

$$u^F(x, y) = 8\sin(ky\sin\alpha)\cos(kx\cos\alpha)\,. \tag{10}$$

Employing the plane-wave expansion [5], Eq. (10) can be re-expressed as

$$u^F(\xi, \eta) = 8\sum_{n=0}^{\infty}(-1)^n ce_{2n}(\eta, q)ce_{2n}(\alpha, q)Mc_{2n}^{(1)}(\xi, q)\,, \tag{11}$$

where $q = (ak)^2/4$, $Mc_{2n}^{(1)}(\cdot)$ are the radial Mathieu functions of the first kind [1], and $ce_{2n}(\cdot)$ are the even angular Mathieu functions [4].

The scattered field u^S may be expressed as

$$u^S(\xi, \eta) = \sum_{n=0}^{\infty} A_n ce_{2n}(\eta, q)Mc_{2n}^{(3)}(\xi, q) \tag{12}$$

where $Mc_{2n}^{(3)}(\cdot)$ are the Mathieu-Hankel functions [1,5] and the complex expansion coefficients A_n are unknown.

The resultant wavefield displacement u that is composed of the free wavefield and the scattered wavefield is given by

$$u(\xi, \eta) = u^F(\xi, \eta) + u^S(\xi, \eta)\,. \tag{13}$$

Applying Eq. (4) and the orthogonal property of angular Mathieu functions leads to

$$A_n = -8(-1)^n ce_{2n}(\alpha, q)\frac{Mc_{2n}^{(1)\prime}(\xi_0, q)}{Mc_{2n}^{(3)\prime}(\xi_0, q)}\,, \tag{14}$$

in which the primes stand for differentiation with respect to the argument ξ.

Substituting Eq. (14) into (12) and then joining with Eq. (11), the derivation of an exact analytical solution is completed.

2.2. Method 2

Alternatively, the application of the method of images is effective. Consider a shallow semi-elliptical canyon (see Fig. 2b). The exact closed-form expression of the displacement field had been derived explicitly by Wong and Trifunac [7], that is,

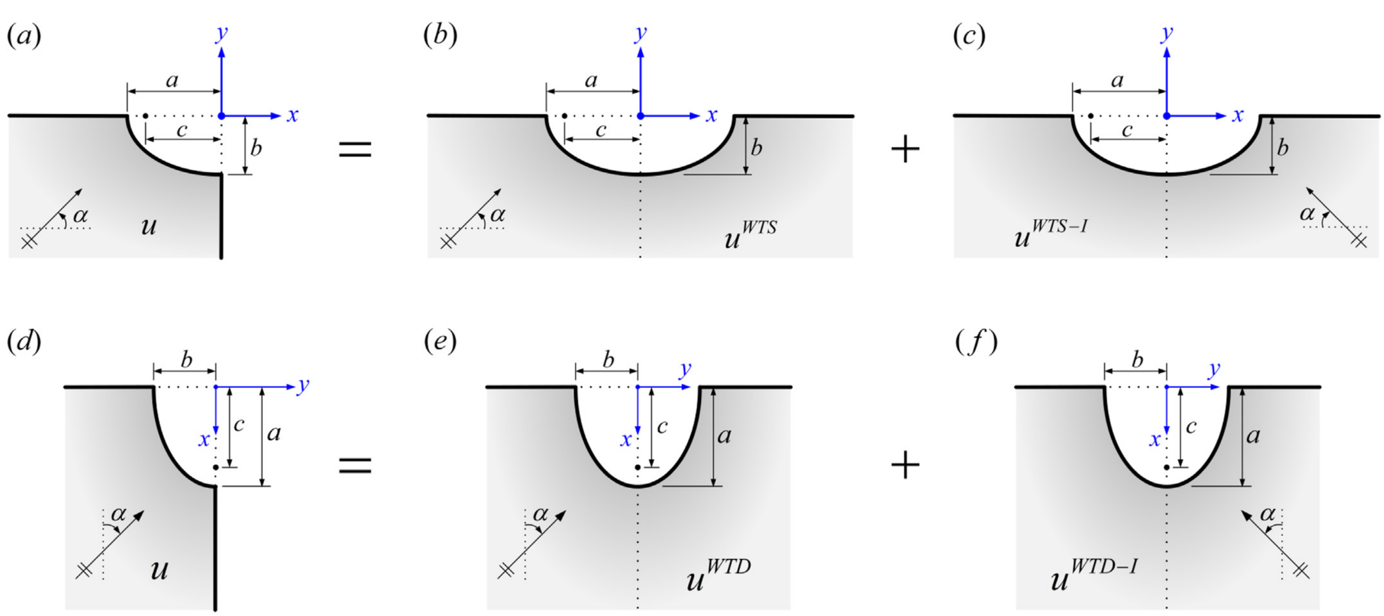

Fig. 2. Principle of superposition applied to construct solutions to antiplane scattering problems. (a)–(c) Shallow case. (d)–(f) Deep case.