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Time-harmonic loading over transversely isotropic and layered elastic half-spaces with imperfect interfaces

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ABSTRACT

In this study, we investigate the time-harmonic response of transversely isotropic and layered half-spaces with imperfect interfaces. The solution in each layer is expressed in terms of the cylindrical system of vector functions so that the normal (dilatational) and shear (torsional) deformations can be separated and solved in terms of the LM- and N-types of the vector function system. A new method called DVP (dual variable and position method) is then utilized to solve the corresponding layered problem. Multiple loads in both vertical and horizontal directions over circular regions on the surface of the layered half-space can be applied and multiple imperfect interfaces can be assumed to model the real interface condition. Several imperfect interface models, including the direct thin-layer, spring, and density models, are introduced and their equivalence, difference, as well as their effects on the response of the layered half-space are investigated. The formulation is validated with existing solutions and is further applied to study the time-harmonic responses of various layered half-spaces. Numerical results demonstrate clearly the influence of the input frequency, thin interlayer modulus, and the relative distance between the field point and the imperfect interface location on the time-harmonic response of layered half-spaces.

1. Introduction

Analysis of soil-structure interaction is important in safe design of various civil structures and foundations. Besides the traditional finite element methods, the boundary element methods (BEMs) have been also proposed in elastodynamic analysis of civil structures. The BEMs have the advantage of discretizing on the problem boundaries only as compared to any domain discretization approach [1,2]. However, in applying the BEM to the structure dynamics problem, the corresponding fundamental solutions are required [3].

Compared to the anisotropic elastostatics, fundamental solutions or Green's functions in either transient or time-harmonic anisotropic elastic solids are much more difficult to derive [4]. By introducing two displacement potentials, Rahimian et al. [5] derived analytically the fundamental time-harmonic solutions due to any surface loading over a transversely isotropic elastic half-space. Displacements and stresses due to a set of time-harmonic circular ring loads within a transversely isotropic half-space were obtained by Rajapakse and Wang [6]. The transient fundamental solutions for a transversely isotropic half-plane or half-space were also derived by Wang and Rajapakse [7].

Layered structures are widely used in various engineering fields, particularly in civil engineering as related to pavement structures and foundations. For a layered half-space with material transverse isotropy, the analytical layer-element method was introduced and applied to solve the time-harmonic response by Ai et al. [8] and Ai and Li [9]. When layers with different material properties are combined together, the bond conditions between layers are important. Previous studies showed that road performance could be seriously affected by the bonding condition between layers [10–12]. Hence, characterizing the interface bonding condition and predicting its effect on the pavement/foundation response are very important as these could help us to design the pavement/foundation for a longer lifetime.

Several models were proposed to solve the bonding problems between different layers. The thin-layer model is one of the commonly used ones. An imperfect interface (where the displacements and/or tractions are discontinuous) between different layers can be generally modelled as a continuous thin layer with a very small thickness. On the other hand, a true thin layer with different material properties could be also represented "equivalently" by an imperfect interface with both normal and shear interface moduli. In terms of these interface moduli, the continuity conditions of the traction and displacement vectors across the interface between the two adjacent layers can be modelled

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List of Symbols

c_{ij}	Elastic stiffness coefficients
h_j	Thickness of layer j
j	Layer index
k_r, k_z	Interface moduli in horizontal and vertical directions
m	Order of Bessel function
(p_x, p_y, p_z)	Magnitudes of surface load in x-, y- and z-directions
(r,θ, z)	Variables in cylindrical coordinate system
$\boldsymbol{t}$	Traction vector
$\boldsymbol{u}$	Displacement vector
u_i	Displacement components in i-direction
E_h, E_v	Young's moduli in horizontal and vertical directions
$\boldsymbol{N}^j$	Propagating matrix $\boldsymbol{N}$ for N-type in layer j
$\boldsymbol{I}$	Identity matrix
P_h	Magnitude of horizontal load
P_v	Magnitude of vertical load
R	Radius of surface load circle
$J_m(\lambda r)$	Bessel function of order m
$\boldsymbol{S}^j$	Propagating matrix $\boldsymbol{S}$ for LM-type in layer j
T_L, T_M, T_N	Expansion coefficients of traction in transform domain
U_L, U_M, U_N	Expansion coefficients of displacement in transform domain
α_r, α_z	Interface compliances in horizontal and vertical directions
β	Material damping factor
ε_{ij}	Strain tensor
λ	Variable in transform domain
μ_h, μ_v	Shear moduli in horizontal and vertical directions
ν_h, ν_v	Poisson's ratios in horizontal and vertical directions
ρ	Material density
σ_{ij}	Stress tensor
ω	Angular frequency $= 2\pi f$
ω_0	Dimensionless angular frequency
ϕ	Load angle

[13]. A simple and yet most commonly used one among various imperfect interface models is the spring model by Goodman et al. [14]. In terms of the spring model, only the displacement vector is discontinuous across the interface between the adjacent layers. A further simplified model was proposed by Hakim et al. [15] where the imperfection is only in the horizontal shearing direction whilst the normal direction is well bonded. In studying wave propagation in layered systems, another typical model of imperfection is also introduced where the traction vector is discontinuous across the interface whilst the displacements are continuous. The traction discontinuity relation can be derived from a given thin layer and the result is proportional to the density in the thin layer [16]. Therefore, this imperfect interface model is called density model. In most previous pavement analyses [17–19], only shear imperfection was assumed (i.e., only the horizontal displacement component was discontinuous).

In this paper, the general time-harmonic surface loading over an isotropic/transversely isotropic (ISO/TI) layered half-space with perfect/imperfect interface conditions is studied. Different imperfect bonding models are considered, including the direct thin-layer model, spring models (with different interface moduli in different directions) and the density model. A novel propagator matrix method (i.e., the dual variable and position method, or DVP) is further introduced, which is more accurate and suitable for layered structures with thin layers under high frequency excitation. This paper is organized as follows: In Section 2, we describe the problem and introduce the basic equations. In Sections 3 and 4, we derive the time-harmonic solutions in terms of the cylindrical system of vector functions and the DVP method. In Section 5, the accuracy and correctness of our method are first validated against existing results and new examples with potential applications are then presented. Conclusions are drawn in Section 6.

2. Description of the boundary value problem

We consider a layered structure made of n-layers over a homogeneous half-space with each layer being made of different TI materials (Fig. 1). A cylindrical coordinate system (r,θ, z) is attached to the layered structure so that the (r,θ)-plane is on the surface and the layered half-space is in the positive z-region. Let layer j be bonded by its lower interface at z_j and upper interface at z_{j-1} with thickness $h_j = z_j - z_{j-1}$. It is obvious that $z_0 = 0$ and z_n. Across each interface, displacement and traction vectors are assumed to be continuous, except for the imperfect interface to be presented in detail later on. Without loss of generality, we also assume that a time-harmonic load in either vertical or horizontal direction, proportional to $e^{i\omega t}$ (with $\omega = 2\pi f$ being the angular frequency), is applied on the top surface of the layered half-space at $z_0 = 0$. As such, all field quantities are proportional to $e^{i\omega t}$. For instance, the solution of the displacement vector can be expressed as $\boldsymbol{u}(r,\theta,z;t) = \boldsymbol{u}(r,\theta,z;\omega)e^{i\omega t}$. Thus, we solve only the proportional factor of the displacement vector $\boldsymbol{u}(r,\theta,z;\omega)$ (and also for the strains and stresses) which is independent of t but dependent of ω, with the understanding

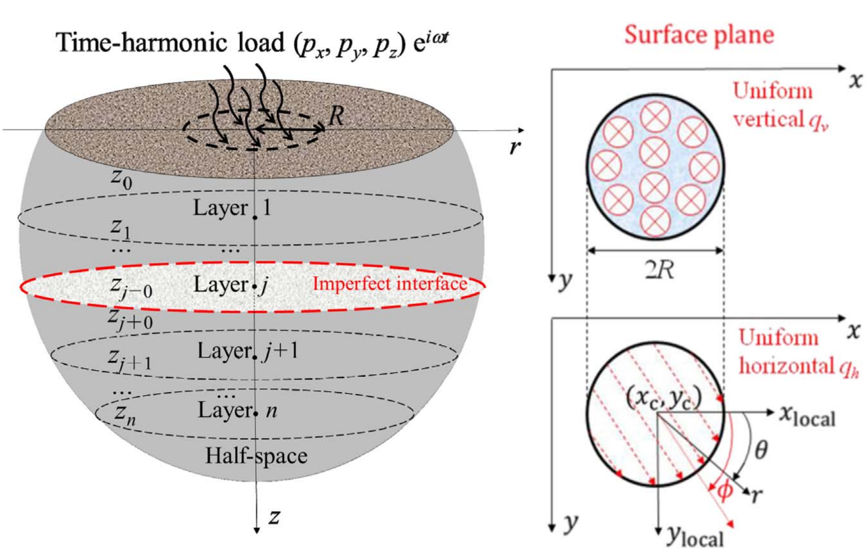

Fig. 1. A TI elastic and layered half-space under a general time-harmonic surface load (in both vertical and horizontal directions) within the circle of radius R. Interface z_j between layers j and $j+1$ is imperfect with values on its upper and lower sides being indicated by -0 and $+0$.