Soil Dynamics and Earthquake Engineering 109 (2018) 150–153

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Dynamic response of an infinite beam resting on a Winkler foundation to a load moving on its surface with variable speed

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ARTICLE INFO

Keywords:
Infinite beam
Winkler foundation
Moving load
Variable speed
Laplace transform
Fourier transform

ABSTRACT

The problem of the dynamic response of an infinite beam resting on a Winkler foundation to a load moving on its surface with variable speed is solved here analytically/numerically under conditions of plane strain. The beam is linearly elastic with viscous damping and obeys the theory of Bernoulli-Euler. The elastic foundation is characterized by its spring constant and hysteretic damping coefficient. The moving point load has an amplitude harmonically varying with time and moves with constant acceleration or deceleration along the top beam surface. The problem is solved by first applying the Fourier transform with respect to the horizontal coordinate x and the Laplace transform with respect to time t to reduce the governing equation of motion of the beam to an algebraic one, which is solved analytically. The transformed beam deflection solution is inverted numerically after some simplifying analytical manipulations to produce the time domain beam response. Parametric studies are conducted in order to assess the effects of the various parameters on the response of the beam, especially those of acceleration and deceleration. Comparisons with the case of a finite beam are also done in order to assess the effect of the beam length.

1. Introduction

The simplest possible model for a rigid pavement under moving vehicle loads is that of an elastic beam or plate on Winkler elastic foundation [1]. The beam or plate can be finite or infinite, thin or thick, with or without viscous damping and the Winkler foundation can consist of vertical and/or horizontal springs and zero or nonzero damping of the viscous or hysteretic type. One can mention here the works of Thompson [2], Achenbach and Sun [3], Sun [4,5], Kim and Roesset [6], Basu and Kameswara Rao [7] and Yu and Yuan [8], dealing with an infinite beam and the one by Lee [9] dealing with a finite beam.

All the existing works utilizing the above models are restricted to the case of loads moving with constant speed. Consideration of constant speed though, does not fully reflects reality, since vehicle loads usually move with speed varying with time.

Very recently, Beskou and Muho [10], were able to study the effect of variable speed on the response of a finite beam on a Winkler foundation to moving loads analytically. The problem was solved in [10] by modal superposition and computation of the resulting Duhamel's integral numerically. This method is applicable only in cases the beam is finite with well-defined boundary conditions at its two ends so as to express its lateral deflection as a superposition of its modal shapes. In cases where the beam is of infinite extend, this method is not applicable. The method of using a moving coordinate system to eliminate the time and reduce the problem to an ordinary differential equation which can be easily solved (e.g., in [2,3,7]), is restricted to the case of loads moving with constant speed.

In the present work, the problem of an infinite beam resting on a Winkler foundation and subjected to a load moving with variable speed on its top surface is solved analytically/numerically by extending the method of Yu and Yuan [8] from the constant speed case to the variable speed case.

The method employs Fourier and Laplace transforms with respect to the horizontal coordinate x and the time t, respectively, to reduce the governing equation of motion of the beam to an algebraic one, which is easily solved analytically. Then the transformed beam deflection solution is inverted numerically after some simplifying analytical manipulations to produce the time domain beam response. This method is similar to the one employing a double Fourier transform with respect to both x and t described in Kim and Roesset [6] but leads to a much simpler transformed solution than the one of [6]. Thus, the present method requires the numerical evaluation of simpler integrals than in [6] where the double inverse fast Fourier transform is used.

Damping is considered for both the beam and the foundation, while the point load maybe constant or varying harmonically with time. Extensive parametric studies are performed to assess the various

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https://doi.org/10.1016/j.soildyn.2018.02.034
Received 14 January 2018; Received in revised form 25 February 2018; Accepted 25 February 2018

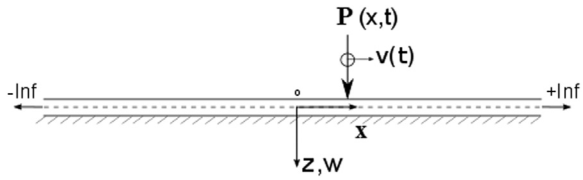

Fig. 1. Infinite beam on Winkler foundation with moving load.

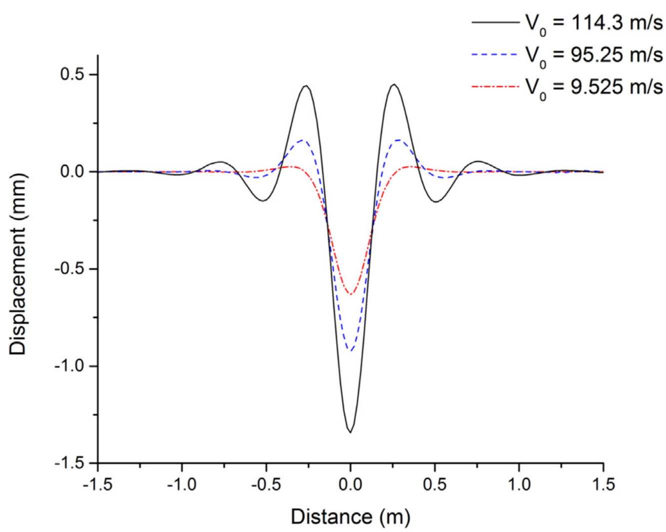

Fig. 2. Beam displacement under the load versus distance along the x axis for zero damping.

Table 1
Dimensionless maximum deflection $\overline{w}$*10^5 of an infinite beam on elastic foundation under moving accelerating load for various parameters.

V_0 (m/s)	a (m/s^2)	k_f (MPa)	c (kNs/m)	Infinite Beam (1)	Finite Beam l = 50 m (2)	[(1)-(2)]/1 (%)
0	0	–	–	–	–	
	6	40	500	0.826504	0.772461	6.54%
			1500	0.748733	0.736073	1.69%
		120	500	0.364954	0.313203	14.18%
			1500	0.358306	0.310275	13.41%
	12	40	500	0.818648	0.769446	6.01%
			1500	0.693399	0.703747	−1.49%
		120	500	0.365607	0.313301	14.31%
			1500	0.350829	0.307532	12.34%
20	0	40	500	0.828043	0.771379	6.84%
			1500	0.765709	0.723657	5.49%
		120	500	0.365825	0.31324	14.37%
			1500	0.359809	0.309321	14.03%
	6	40	500	0.819550	0.768431	6.24%
			1500	0.706256	0.693172	1.85%
		120	500	0.366001	0.313331	14.39%
			1500	0.351924	0.306605	12.88%
	12	40	500	0.810274	0.765529	5.52%
			1500	0.660133	0.667097	−1.05%
		120	500	0.366068	0.313404	14.39%
			1500	0.343859	0.303978	11.60%
40	0	40	500	0.810215	0.759587	6.25%
			1500	0.649872	0.622355	4.23%
		120	500	0.365272	0.313583	14.15%
			1500	0.343529	0.298929	12.98%
	6	40	500	0.804898	0.756814	5.97%
			1500	0.614450	0.604659	1.59%
		120	500	0.362642	0.313679	13.50%
			1500	0.338370	0.296573	12.35%
	12	40	500	0.793924	0.754102	5.02%
			1500	0.583276	0.588641	−0.92%
		120	500	0.365961	0.313759	14.26%
			1500	0.330183	0.294285	10.87%

Table 2
Dimensionless maximum midspan deflection $\overline{w}$*10^5 of infinite beam on elastic foundation under moving decelerating load for various parameters.

V (m/s)	a (m/s^2)	k_f (MPa)	c (kNs/m)	One axle (1)	Finite Beam l = 50 m (2)	[(1)-(2)]/1 (%)
20	−6	40	500	0.832028	0.774418	6.92%
			1500	0.799478	0.760616	4.86%
		120	500	0.365800	0.313152	14.39%
			1500	0.363572	0.312144	14.15%
	−12	40	500	0.831555	–	–
			1500	0.804197		
		120	500	0.366011	–	–
			1500	0.363311		
40	−6	40	500	0.818187	0.762516	6.80%
			1500	0.692825	0.642180	7.31%
		120	500	0.365432	0.313504	14.21%
			1500	0.350562	0.301334	14.04%
	−12	40	500	0.823309	0.765363	7.04%
			1500	0.720596	0.664572	7.77%
		120	500	0.364849	0.313420	14.10%
			1500	0.355099	0.303884	14.42%

problem parameters on the response. Comparisons with the case of the finite beam of [10] with very large length are also made in the framework of validation studies.

2. Statement and solution of the problem

Consider an infinite Bernoulli-Euler beam resting on a Winkler elastic foundation and subjected to a concentrated load P(x,t) moving on its surface with a variable speed V(t), as shown in Fig. 1. The equation of lateral motion of this beam has the form

$$EIw''''(x, t) + kw(x, t) + c\dot{w}(x, t) + m\ddot{w}(x, t) = P(x, t) \quad (1)$$

where w = w(x,t) is the lateral deflection of the beam, EI is the flexural rigidity of the beam, k is the foundation spring constant, $c = c_b + c_f$ is the damping coefficient with c_b corresponding to the beam and c_f to the foundation, m is the beam mass per unit length, primes and overdots denote differentiation with respect to the horizontal coordinate x and time t, respectively and P(x,t) is the moving concentrated (point) load. This load can be expressed as

$$P(x, t) = P_o\delta(x - x_o) \quad (2)$$

where P_o is its constant magnitude and x_o is expressed in terms of the initial velocity V_o and the constant acceleration (with + sign) or deceleration (with – sign) α as

$$x_o = V_{ot} \pm \frac{1}{2}at^2 \quad (3)$$

Initial conditions are assumed to be zero and w and its derivatives with respect to x tend to zero as x approaches $\pm\infty$.

Following the approach in [8], the Fourier transform with respect to x and the Laplace transform with respect to t are applied on Eq. (1) to reduce this partial differential equation into an algebraic one, which can be easily solved analytically. These two transforms for a function f (x,t) are defined as

$$\overline{f}(\xi, t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x, t)e^{i\xi x}dx \quad (4)$$

$$\tilde{f}(x, s) = \int_0^{\infty} f(x, t)e^{-st}dt \quad (5)$$

Application of Fourier transform with respect to x on Eq. (1) and use of Eq. (2) result in

$$EI\xi^4\overline{w}(\xi, t) + k\overline{w}(\xi, t) + c\overline{\dot{w}}(\xi, t) + m\overline{\ddot{w}}(\xi, t) = P_o e^{-i\xi x_0} \quad (6)$$

Application of Laplace transform with respect to t on Eq. (6)