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Variability of shallow soil amplification from surface-wave inversion using the Markov-chain Monte Carlo method

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ABSTRACT

We conduct numerical experiments to estimate the variability of 1D linear and nonlinear soil amplifications due to the uncertainty in shallow S-wave velocity profiles derived from surface-wave phase velocity inversions using the Markov-chain Monte Carlo method. We first generate synthetic, observed phase velocities of Rayleigh waves for two- and three-layer models of shallow soil. Our final models from sampling can explain well the true S-wave velocity profiles and the phase velocities. We also estimate the uncertainties of each model parameter. A synthetic strong motion is applied to the engineering bedrock of the sampled models to obtain the surface motion assuming linear and nonlinear amplifications. It is found that the nonlinear amplification shows less variability and also has a flatter spectral shape than the linear amplification, particularly at high frequencies. The distributions of ground motion proxies generally have less uncertainty for the nonlinear amplification as well. We also find that the observational errors of the phase velocities have less influence on the variability of the nonlinear amplification than the linear case. This result is caused by the high damping factor applied in the nonlinear soil response.

1. Introduction

The properties of seismic waves such as spectral shape and amplitude can be altered during seismic wave propagation in near-surface layers. Soil amplification is used to quantify the alteration of seismic waves due to local site effects. It is known that the S-wave velocity profile of a soil mostly controls the soil amplification. Therefore, the S-wave velocity profile at an area of engineering interest must be known. Geophysical exploration surveys are widely employed to deduce S-wave velocity profiles. In general, we can categorize such surveys based on their use of active or passive methods. Exploration via multichannel analysis of surface waves (or MASW) [1] is commonly used as an active source measurement, while microtremor exploration [2] is employed for a passive measurement. These two methods, known as surface-wave techniques, have been established using surface-wave dispersions to derive an S-wave velocity model.

The surface-wave techniques are based on estimations of the frequency-dependent phase velocity of surface waves, which are mainly Rayleigh waves. Many techniques have been proposed in the literature for the inversion of Rayleigh wave phase velocity into an S-wave velocity profile, such as the least-square and heuristic methods. Least-square methods [3,4] are conventionally used in phase velocity inversions. However, these approaches are well known to have numerical instability and experience trapping at local minimum misfits. Some of these practical difficulties can be avoided using heuristic methods [5]. Genetic algorithms and simulated annealing are two heuristic methods that have been widely applied in geophysical inversions, including surface-wave inversions [5]. Nevertheless, these approaches cannot estimate the model resolution directly, which can be done easily using the conventional least-square method.

During surface-wave measurement, observational error in the surface-wave phase velocity is inevitable. The error may arise from noise in recorded signals or the existence of higher modes. Yamanaka [6–8] showed that the observational errors in phase velocity were linearly related to the uncertainty in S-wave velocity profiles. Since the S-wave velocity profile mainly affects soil amplification, the uncertainty in the S-wave velocity profile may also cause inaccuracy in ground motion estimations in engineering design. Hence, it is important to understand the propagation of uncertainty in the S-wave velocity profile derived from surface-wave inversions and its effect on the variability of the soil amplification.

Some studies [6–11] have been devoted to understanding the effects of the uncertainty in S-wave velocity profiles on 1D soil responses. They attained the uncertainty in S-wave velocity profiles from inversions of surface-wave phase velocity, and then applied it to estimate the variation of the corresponding soil response. We refer to the 1D soil

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response in the studies by Foti et al. [10] and Jakka et al. [11] as nonlinear amplification in our discussion, because they use the equivalent linear method. We also assume that their soil models follow shear modulus and damping factor curves. Previous studies have suggested that the uncertainty in the S-wave velocity profiles play an important role in the variation of linear amplification of seismic motion [6–9]. Jakka et al. [11] reported that the resolution of an S-wave velocity profile could lead to significant effects on seismic nonlinear responses. However, Foti et al. [10] noticed that the uncertainty in S-wave velocity profiles had a negligible effect on the variations of the linear and nonlinear responses. It seems that there is still no consensus on the effect of the uncertainty in S-wave velocity profiles on the variation of soil response. Thus, we will, in a later section, discuss the reasons for these differences in results.

In this study, we conducted numerical experiments to determine the uncertainty in S-wave velocity profiles using sampled models from inversions of Rayleigh-wave phase velocities using the Markov-chain Monte Carlo (MCMC) method. Then, we used the sampled models to estimate the variations of 1D linear and nonlinear amplifications for comparison. The MCMC method is a heuristic approach that has advantages over the least-squares and other heuristic methods in the inversion of phase velocity. Sampled models in the MCMC inversion can be used not only to estimate the uncertainty in S-wave velocity profiles, but also to determine the variation of 1D linear soil amplification [6–9]. However, many studies suggest that the soil will behave nonlinearly during strong shaking with a peak ground acceleration (PGA) exceeding 100 cm/s^2 [12,13]. Hence, our main goal is to understand the effect of the uncertainty in S-wave velocity profiles from phase velocity inversions on the variability of the nonlinear soil amplification.

We used an equivalent linear method (SHAKE91) [14,15] to approximate a nonlinear soil response. Nonlinear models can be assumed in order to explain soil behavior with respect to effective stresses during cyclic loading, which is not included in SHAKE91 [16]. Nonetheless, the equivalent linear method has still been widely employed in the engineering community, owing to its simplicity and robustness. Moreover, many studies have shown that this method gives reasonable results for the estimation of nonlinear soil response as compared to nonlinear methods, for moderate strain levels [17,18]. It is known that the equivalent linear method may have significant over-damping effects at high frequencies if the strain level is too large, particularly for soft soil [19,20]. Thus, we must carefully consider the strain level and type of soil during seismic response analysis using SHAKE91.

2. Methods

2.1. MCMC inversion

The MCMC inversion samples models based on misfit values $E(\boldsymbol{m})$ for a model parameter $\boldsymbol{m}$. The model parameters denote the S-wave velocity (Vs) and thickness (H) of a layered model in this study. The misfit value is defined as:

$$E(\boldsymbol{m}) = \sum_{i=1}^{n}\left[\frac{O(f_i) - C(f_i)}{\sigma(f_i)}\right]^2, \tag{1}$$

where $O(f_i)$, $C(f_i)$, and $\sigma(f_i)$ are the observed phase velocity, the calculated phase velocity, and the standard deviation of the observed phase velocity at the frequency f_i, respectively. Additionally, n is the number of observed phase velocities.

The calculation of theoretical phase velocity for a horizontally layered model in the MCMC inversion is estimated using Haskell's method [21] assuming the fundamental mode. Besides thickness and S-wave velocity, it also requires information on density (ρ) and P-wave velocity (Vp), which are less sensitive to phase velocity than the thickness and S-wave velocity. Moreover, we used the following empirical relationship [22]:

$$Vp = 1.11Vs + 1290, \tag{2}$$

to calculate P-wave velocity from S-wave velocity (m/s). This relation has been used in several previous studies [5–8]. The density of each layer is given in advance.

In the MCMC method, the posterior probability distribution $p(\boldsymbol{m}|\boldsymbol{d})$ is formulated based on Bayes' theorem for the model parameter $\boldsymbol{m}$ and the given data $\boldsymbol{d}$. The given data denote the observed phase velocity. The theorem is expressed as:

$$p(\boldsymbol{m} \mid \boldsymbol{d}) = \frac{p(\boldsymbol{d} \mid \boldsymbol{m})p(\boldsymbol{m})}{p(\boldsymbol{d})}. \tag{3}$$

Here, $p(\boldsymbol{m}|\boldsymbol{d})$ is a conditional probability distribution of the model parameters given the observed phase velocity. Moreover, $p(\boldsymbol{m})$ and $p(\boldsymbol{d})$ are the prior probability distribution of the model and the probability distribution of the data, respectively. We assume that the distribution of the data and the prior distribution of the model parameters are constant, because we typically have the data before the inversion and we do not have prior information on the model parameters. The $p(\boldsymbol{d}|\boldsymbol{m})$ is a conditional probability distribution related to a likelihood function $L(\boldsymbol{m})$, and it can be written as:

$$p(\boldsymbol{d} \mid \boldsymbol{m}) \propto L(\boldsymbol{m}) = exp[-E(\boldsymbol{m})]. \tag{4}$$

We regard the posterior distribution of model parameters as a solution of our inversion considering their uncertainties. From the above assumption, we can write the posterior distribution as:

$$p(\boldsymbol{m} \mid \boldsymbol{d}) \propto exp[-E(\boldsymbol{m})]. \tag{5}$$

To find a stationary sampling from the posterior distribution in Eq. (5), we apply the Metropolis–Hastings algorithm [23]. This algorithm constructs a Markov chain to estimate a stationary sampling condition of the models. The sampling process in the MCMC method uses a series of iterations to generate a stationary sampling from a random initial model. Therefore, we need to discard sampled data before reaching a stationary state; this is known as a burn-in period. We employ Geweke's convergence criteria [24] to determine the burn-in period by using the Z-value. The Z-value for each parameter is calculated as:

$$Z = \frac{|g_1 - g_2|}{\sqrt{s_1 + s_2}}, \tag{6}$$

where g_1 and s_1 are the average and the variance for the first p data, while g_2 and s_2 are the average and the variance for q data from the end of the iteration. Assuming that the number of data after the burn-in period is L, the p and q were set to be 10% and 50% of L, respectively. We calculated different burn-in periods until the maximum Z-values of all the parameters were less than 1.96. This Z-value shows a stationarity at a significance level of 5%.

2.2. Estimation of soil amplification

After inverting the phase velocity to an S-wave velocity model, we estimated the soil amplification. We used a frequency range of 0.1–10 Hz in this study to avoid prominent over-damping in the high frequency range from the nonlinear soil response calculation by SHAKE91, as mentioned previously. The method computes the transfer function and surface motion in the frequency domain. We define the amplification as a surface to outcrop spectral ratio.

We can determine surface motions from the amplification analysis for the sampled models and an input acceleration wave. Moreover, we can also estimate ground motion proxies of the surface motions such as fundamental frequency, maximum amplification, PGA, peak ground velocity (PGV), and response spectral acceleration.