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Dynamic response of an elastic plate on a cross-anisotropic poroelastic half-plane to a load moving on its surface

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ABSTRACT

The dynamic response of a thin, isotropic and linear elastic plate on a half-space soil medium to a load moving on its surface is analytically obtained under plane strain conditions. The soil is assumed to be homogeneous, cross-anisotropic and water fully saturated linear poroelastic. Anisotropy manifests itself in the elastic skeleton, the pore water pressure and the permeability. The full Biot's equations of motion are employed. The load is assumed to be distributed over a finite length and moves with constant speed. The moving load is expanded in a complex Fourier series form involving the horizontal coordinate, time and speed. All the response quantities referring to the plate and the soil are also expanded in Fourier series in the same way. Thus, the governing partial differential equations of motion for the plate and the poroelastic soil (u-p formulation) are reduced to a system of ordinary differential equations with only one independent variable, the vertical coordinate. Use of compatibility and equilibrium at the plate-poroelastic soil interface as well as the boundary conditions of the problem, result in the solution of the above system of equations in analytic form. This solution is validated by comparing Its results against other analytic solutions referring to simpler cases (isotropic elastic with a plate and isotropic poroelastic soil without a plate). Finally, parametric studies are conducted to assess the anisotropy effect on the response of the system to moving loading for various values of porosity, permeability and load speed.

1. Introduction

Determination of the dynamic response of flexible and rigid road pavements to moving vehicle loads is a very important problem in pavement analysis and design. An extensive literature review on the subject of dynamic effects of moving loads on road pavements has been recently compiled by Beskou and Theodorakopoulos [1]. The existing works (181 of them) on the subject have been classified in that review in accordance with the modeling of the pavement structure (e.g. a plate on a layered half-space), the modeling of the material behavior of the pavement and the supporting soil, the modeling of the moving load and the methods of solutions of the pertinent dynamic problem.

Concerning the methods of solution, one can broadly group them into two categories: analytical and numerical methods. Numerical methods, such as the finite element method (FEM), can handle complex geometries and material behavior [2,3], while analytical methods are restricted to simple geometries and material behavior [4,5]. However, the latter methods are very useful because through their solutions, usually in closed form, provide a physical insight into the problem, permit an easy performance of parametric studies and serve as benchmarks for the assessment of the accuracy of numerical methods. Analytical methods of solution are associated with linear material behavior for the pavement and the supporting soil. This behavior can be linear elastic, linear viscoelastic or linear poroelastic with isotropic or anisotropic, homogeneous or nonhomogeneous (layered) character.

Pavement layers are in reality cross-anisotropic (or transversely isotropic) i.e., they have one axis of symmetry (the vertical axis perpendicular to the plane of the pavement) and thus their behavior to any plane orthogonal to that axis is isotropic. This anisotropy which can be determinate by laboratory tests [6,7], has been found by numerical methods (FEM) [8,9] to significantly affect the pavement response to moving vehicle loads.

In a very recent work of the authors [10], the effect of soil cross-anisotropy on the response of a rigid pavement (elastic plate) resting on that soil to a moving load on its surface, has been studied analytically under conditions of plane strain. In the present work, the same effect is also studied analytically under plane strain conditions for the case of an elastic plate resting on a cross-anisotropic fully saturated with water poroelastic soil.

For this reason, only analytical works on the subject of the present

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paper will be reviewed in the following with emphasis on those works, which are more closely related to the present work dealing with poroelastodynamics, in an effort to establish originality of this work. For more information on the subject, one can consult the previous work of the authors [10] dealing with elastic anisotropic soil behavior, the aforementioned review study [1] and the two classical papers of Biot [11,12] as well as the very recent excellent book of Cheng [13] on isotropic and anisotropic poroelastodynamics.

The existing analytic works closely related to the present work, can be grouped into the following four categories:

1. Works dealing with homogeneous or layered, isotropic, poroelastic half-plane or half space under dynamic load on its surface acting directly or through a rigid or flexible plate [14–21]. In all cases harmonic time variation is assumed and the problem in the frequency domain consists of partial differential equations with (x, z) and (r, θ, z) coordinates in two and three-dimensions, respectively. If there is axisymmetry, then θ is eliminated. If not, series expansions in terms of cosnθ are employed. Fourier transform with respect to x or Hankel transform with respect to r reduce the problem to the solution of ordinary differential equations with respect to z, which can be easily obtained. In some references [14,15] the equations are simplified by use of Helmholtz decomposition before the application of transforms.
2. Works dealing with homogeneous or layered, isotropic, poroelastic half-space under point of distributed, constant in amplitude moving with constant speed loads [24–33]. The problem is solved by using Fourier transform (single, or double, or triple) with respect to the variables of time t and the two horizontal coordinates x and y [28–33], or complex Fourier series involving the time t, the speed V and the coordinate x (axis of load motion) [25–27], or moving coordinates to eliminate time and then use Fourier transform with respect to the horizontal coordinate. It should be noticed that references [24,27] do not use the full Biot's equations, but an approximate version of them, which simplifies the problem without affecting accuracy very much.
3. Works dealing with elastic beams or plates on a layered, isotropic, poroelastic half-space under line or distributed, constant in magnitude, moving with constant velocity loads [22,23]. The problem is solved by using Fourier transform (triple) with respect to t, x and y. Equilibrium and compatibility at the structure-soil interface helps to connect the components of the system [22,23].
4. Works dealing with homogeneous orthotropic or transversely isotropic, poroelastic half-plane or half-space under dynamic forces in its interior or its surface [34,35]. Both papers do not utilize the full dynamic equations of poroelasticity in order to simplify the problem. Thus, reference [34] assumes zero inertia terms and easily solves by the operator theory the resulting uncoupled static solid-fluid problem, while reference [35] assumes zero relative acceleration between solid and fluid and harmonic with time forces and solves the problem by Fourier and Hankel transforms.

From the above, one can easily conclude that the case of an elastic beam or plate on a cross-anisotropic (transversely isotropic) poroelastic half-space or half-plane subjected to moving with constant speed loads, i.e., the subject of the present paper, has not been treated as yet in the existing literature. It should be also noticed that with the exception of references [25–27], the analytical solution of all the other references is given in the form of integrals, which have to be evaluated numerically. In this work, the method of expansion in complex Fourier series with respect to x coordinate and time t as applied in references [25–27] is employed and provides the solution in terms of series, which are also evaluated numerically, but in a much easier way than integrals do. Finally, it should be also mentioned that even though the analytical solution of this work actually refers to a soil layer over bedrock, it can be easily used for the case of a half-plane if one uses a very large value for the thickness of that layer.

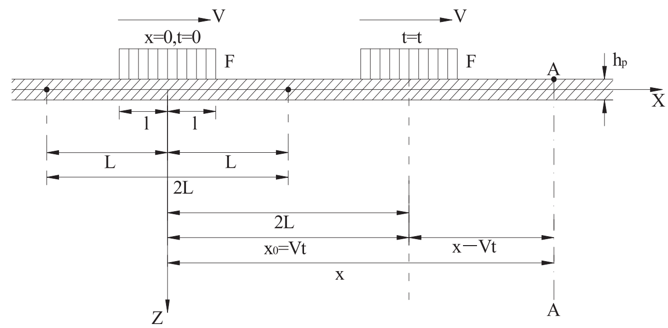

Fig. 1. Geometry of periodic distributed load F moving with constant speed V on the surface of a plate resting on half-plane.

At this point one should emphasize that consideration of anisotropy in poroelastodynamics does only affect the stress-strain equations of the soil skeleton as it is the case with elastodynamics, but the pore water pressure, the permeability and the relative inertia of the poroelastic soil as well. This makes the anisotropic poroelastodynamic problem more difficult than the isotropic elastodynamic one.

2. Statement of the problem

Consider a homogeneous cross-anisotropic (or transversely isotropic) fully water saturated linear poroelastic half-space soil medium supporting a homogeneous isotropic elastic flexural plate under conditions of plain strain, as depicted in Fig. 1 in the framework of the (x, z) coordinate system. It is assumed that a uniformly distributed vertical strip load F(x, t) moves on the top surface of the plate along the x direction with constant speed V as shown in Fig. 1. The goal of this work is the analytical determination of the dynamic response of the plate-soil system (deflection, stresses and porowater pressure of soil and deflection and bending moment of plate) to this moving load and the assessment of the effects of the various parameters of the problem (load, speed, porosity, permeability and especially cross- anisotropy) on that response. This will be accomplished through extensive parametric studies. It is anticipated that the obtained information on the dynamic behavior of this plate-soil system modeling rigid pavements will help engineers to better understand that behavior and improve the design of this category of pavements.

The equations of motion of the cross-anisotropic poroelastic soil medium of Fig. 1 are of the form [13]

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{44}\frac{\partial^2 u}{\partial z^2} + (C_{13} + C_{44})\frac{\partial^2 w}{\partial x \partial z} - \alpha_1\frac{\partial p}{\partial x} = \rho\ddot{u} + \rho_f\ddot{r}_x \tag{1}$$

$$C_{33}\frac{\partial^2 w}{\partial z^2} + (C_{13} + C_{44})\frac{\partial^2 u}{\partial x \partial z} + C_{44}\frac{\partial^2 w}{\partial x^2} - \alpha_3\frac{\partial p}{\partial z} = \rho\ddot{w} + \rho_f\ddot{r}_z \tag{2}$$

$$-\frac{\partial p}{\partial x} - \rho_f\ddot{u} = \frac{\eta}{k_x}\dot{r}_x + m^*\ddot{r}_x \tag{3}$$

$$-\frac{\partial p}{\partial z} - \rho_f\ddot{w} = \frac{\eta}{k_z}\dot{r}_z + m^*\ddot{r}_z \tag{4}$$

where u = u(x, z, t) and w = w(x, z, t) are the displacements of the soil skeleton along the x and z directions, respectively, p = p(x, z, t) is the porewater pressure, overdots indicate differentiation with respect to time t, r_x and r_z are defined as.

$$r_x = \varphi(u^f - u), \quad r_z = \varphi(w^f - w) \tag{5}$$

η is the dynamic viscosity of the fluid, k_x and k_z the permeabilities along