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Dynamic behaviour of an infinite saturated transversely isotropic porous media under fluid-phase excitation

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ABSTRACT

One of the required fundamental solutions of a poroelastodynamic problem is the solution of the saturated porous medium under a fluid flux or pressure. In the framework of coupled formulations of the displacement and pore-fluid pressure proposed by Biot, the fundamental solutions for a transversely isotropic fluid-saturated porous full-space are analytically presented when the domain of interest is affected by a time-harmonic fluid either ring flux or ring pressure. To this end, a solely scalar potential function is used to uncouple the Biot's coupled partial differential equations, where a sixth order partial differential equation achieved, from which the potential function is determined with the aid of Hankel integral transform and Fourier expansion series. The analytical fundamental displacements, stresses and pore-fluid pressure are presented in the form of one-dimensional semi-infinite integrals, which are due to inverse Hankel integral transforms. The integrals are also degenerated for the isotropic material in mechanical point of view. Because of the complexity of the integrands, the integrals are evaluated numerically. To this end, an adaptive numerical quadrature is adopted and coded in MATHEMATICA software. Some numerical results have been provided to illustrate the displacements, stresses and pore-fluid pressure.

1. Introduction

The study of a fluid saturated porous medium (soil) is of fundamental importance to several disciplines of engineering some of which are earthquake engineering, seismology, geotechnical engineering, marine infrastructures, hydrology and geophysics. In this framework, the soil should be considered as a two-phase material, including the solid skeleton and pore-fluid. To model this complex phenomenon, a coupled formulation including the solid-fluid interaction should be established, and theory of poroelasticity refers to coupling between solid displacements, applied stresses and pore-fluid pressure. The standard theory of poroelasticity was firstly introduced by Biot [1–3], who derived coupled partial differential equations that govern both the motion and transport equations for the saturated porous medium. Zienkiewicz et al. [4] derived a simplified formulation of Biot's equation, known as $\mathbf{u} - p$ formulation, in which the relative acceleration of fluid has been neglected. Therefore, the unknown variables are reduced to three solid displacements, defined as displacement vector $\mathbf{u}$, and pore-fluid pressure, p.

To solve the poroelastodynamic problem with boundary element method (BEM), the first and the key point is to derive the fundamental solutions of the problem. The dynamic fundamental solutions for a fully saturated poro1-3us full-space containing isotropic materials have been investigated in previous researches; however, the solutions have been presented for external forces rather than pressure or flux. Burridge and Vargas [5] first published the fundamental solutions for the saturated isotropic porous full-space due to single point load acting on the solid phase. Later, Norris [6] obtained the fundamental solutions for a time-harmonic point force in the solid skeleton as well as a time-harmonic point force in the pore-fluid. Taguchi and Kurashige [7] derived the fundamental solutions for a transversely isotropic poroelastic solid medium under step-like point forces and an instantaneous fluid point source. Zhou et al. [8] obtained the transient solution of saturated soil to a concentrated loading assuming incompressible constitutes while the inertia coupling between the solid skeleton and fluid has been neglected. Chen et al. [9] analyzed a poroelastic half space under time harmonic buried loading and presented the Green's functions in cylindrical coordinate system. Kamalian et al. [10] obtained the three dimensional fundamental solutions for saturated poroelastic problem. They utilized the $\mathbf{u} - p$ formulation of saturated porous media and neglected the compressibility of fluid and solid particles. Pan [11] developed the Green's functions for a layered poroelastic half-space. Lo et al. [12] obtained the analytical solution of poroelastic half-space due to harmonic surface traction, assuming the decoupled poroelasticity

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Nomenclature

a	radius of fluid ring flux
b	radius of fluid ring pressure
C_{ijkl}	elasticity constants
E	Young's moduli in the plane of transverse isotropy
E'	Young's moduli in the direction normal to the plane of transverse isotropy
F	Scalar potential function
G	shear modulus in the plane normal to the axis of symmetry
G'	shear modulus in planes normal to the plane of transverse isotropy
J_m	Bessel function of the first kind and m^{th} order
k_1	intrinsic permeability in any direction in horizontal plane
k_3	intrinsic permeability in vertical direction
K_s	bulk modulus of solid phase
K_f	bulk modulus of fluid phase
M	Biot's modulus
n	porosity
p	pore-fluid pressure
P	magnitude of fluid ring pressure
Q	magnitude of fluid ring flux
r	radial coordinate
$\mathbf{u}$	displacement vector of solid
u_r	displacement component in $r-$ direction
u_θ	displacement component in $\theta-$ direction
u_z	displacement component in $z-$ direction
z	vertical coordinate
α_1	Biot effective stress coefficient in horizontal plane
α_3	Biot effective stress coefficient in vertical plane
$\delta(r)$	Dirac-delta function
η	dynamic viscosity of the fluid
θ	angular coordinate
λ_1, λ_2,λ_3	radicals appearing in general solutions
λ and μ	Lame' constants
ν	Poisson's ratio characterizing the lateral strain response in the plane of transverse isotropy to a stress acting parallel to it
ν'	Poisson's ratio characterizing the lateral strain response in the plane of transverse isotropy to a stress acting normal to it
ξ	Hankel's parameter
ρ^s	mass density of solid
ρ^f	mass density of fluid
ρ	mass density of mixture
$\sigma_{ij}(i, j = r,\ \theta,\ z)$	stress tensor
ω	angular frequency
ω_0	nondimensional frequency

equations of Biot [3]. Hou et al. [13] have presented the three dimensional Green's functions for a bimaterial transversely isotropic saturated porous full-space under a point fluid source with the aid of six harmonic functions with undetermined constants. Sahebkar and Eskandari-Ghadi [14], with use of a set of two complete scalar potential functions have reported time-harmonic response of saturated porous transversely isotropic half-space under both point and patch surface tractions. Recently, Pan et al. [15] have, with expressing the unknown functions in terms of three functions, each of which is a solution of 2D Laplace equation with a scaled coordinate, presented the two dimensional fundamental solutions for fluid-saturated orthotropic medium under a line fluid source applied either in the interior of an infinite poroelastic plane or on the surface of a semi-infinite poroelastic plane.

In the present paper, a saturated porous full-space, which is transversely isotropic in both mechanical and transport points of view with a unique axis of symmetry, is considered as the domain of the boundary value problem and the fundamental solutions of the problem due to fluid ring flux and fluid ring pressure are investigated with the aid of potential function introduced in [14]. Formerly, the authors investigated a poroelastic half-space under surface tractions, while the surface was assumed fully permeable. Therefore, the fluid flux and fluid pressure were forced to be zero at the surface. However, to provide a complete and comprehensive solution with an integral based analytical-numerical method, it is essential that all fundamental solutions are available. Thus, it gave the authors a strong incentive to study the dynamic behaviour of a poroelastic medium under fluid flux and fluid pressure in order to provide the fundamental solutions. The focus of the present paper is to provide an analytical form of the fundamental solution as much as it is possible, and to be readily used in integral based analytical-numerical procedure for more complex boundary value problems. To this end, the full-space is decomposed to two half-spaces, and with the use of Fourier series and Hankel integral transforms in a cylindrical coordinate system a sixth order ordinary differential equation is obtained for the potential function in each half-space, from which the potential functions are determined in the Fourier-Hankel space. The unknown coefficients are determined by satisfying both the regularity and continuity conditions. Eventually, the displacements, stresses, and the pore-fluid pressure are presented with the use of the relations between these functions and the potential function in Fourier-Hankel space. These functions are presented in the real domain in the form of semi-infinite line integrals via the theorem of inverse Hankel integral transforms. In order to illustrate the pattern of displacements, stresses and pore-fluid pressure, some numerical results are presented.

2. Statement of problem

The physical domain of interest is taken to be a homogenous full-space, fully saturated poroelastic transversely isotropic in both mechanical and transport points of view. A cylindrical coordinate system $(r,\ \theta,\ z)$, whose $z-$ axis is depth-wise is attached to the domain as a reference (see Fig. 1). Both the mechanical and hydraulic axes of symmetry of material are assumed parallel to the $z-$ axis. A time-harmonic fluid ring flux of radius a and a fluid ring pressure of radius b are considered to be applied on plane $z = 0$ (see Fig. 1). In the absence of body forces, the coupled equations of time-harmonic motion for a homogenous transversely isotropic saturated porous medium can be written as follows [1]

$$\nabla.\ \boldsymbol{\sigma}^s = -\rho^{11}\omega^2\mathbf{u}^s - \rho^{12}\omega^2\mathbf{u}^f + i\omega b(\mathbf{u}^s - \mathbf{u}^f), \tag{1.a}$$

$$\nabla.\ \boldsymbol{\sigma}^f = -\rho^{22}\omega^2\mathbf{u}^f - \rho^{12}\omega^2\mathbf{u}^s - i\omega b(\mathbf{u}^s - \mathbf{u}^f), \tag{1.b}$$

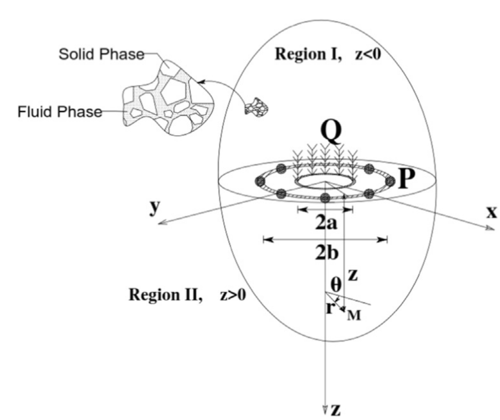

Fig. 1. Saturated porous full-space under fluid ring flux of radius a and fluid ring pressure of radius b.