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Wave dispersion analysis of multi-story frame building structures using the periodic structure theory

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ABSTRACT

Introducing the periodic structure theory in the field of solid-state-physics into the field of civil engineering, this paper conducts a comprehensive theoretical and numerical study on the wave dispersion properties of multi-story frame building structures. At first, the theoretical infinite periodic model for the multi-story frame building structure is developed. Combining the Bloch-Floquet theorem and the State-Space-Transfer-Matrix-Method (SSTMM), the dispersion equation of the infinite periodic system is solved, from which dispersion curves and group/phase velocity curves of a practical multi-story reinforced concrete frame building structure are investigated. Second, a parametric study is performed to investigate the influences of the typical material parameters (density, strength grade of concrete), geometrical parameters (span length, floor height), theoretical simplified parameters as well as the damping parameter of frame building structures on the dispersion relations. Then, harmonic analysis and time history analysis are conducted to analyze the dynamic properties of the finite periodic frame structure in the frequency domain. Numerical results show that, when the main frequency region of the external excitations (like seismic waves) falls into the passing bands, dynamic responses of the multi-story frame building structure are very large; while the main frequency region of the external excitations falls into the attenuation zones, seismic responses of the multi-story frame building structure are very small.

1. Introduction

The propagation of elastic waves in periodic composite structures, named *phononic crystals* or *elastic metamaterials*, has received considerable attention in recent years [1–4]. The most fascinating property of these periodic composite structures is the filtering effect for elastic waves. In detail, in some frequency regions, known as passing bands (PBs), waves/vibrations can propagate in these periodic structures freely; while in other frequency regions, known as stop bands (SBs), waves/vibrations cannot. This interesting property has many potential applications in civil engineering such as isolating seismic vibrations, reducing traffic noise and so on. Sigalas and Economou [1] reported the first dispersion relationship and the first Bragg scattering frequency band gap for elastic waves in a periodic structure consisting of identical spheres placed periodically within a host homogeneous material. After that, a great deal of attention has been paid on the existence of the band gap for elastic waves in Bragg scattering periodic structures (BSPSs). Later, it was found that the spatial modulation of the elasticity of the BSPSs must be of the same order as the wavelength in the gap. Thus, in order to prevent lower-frequency vibrations, the size of the BSPS must be very large, which is not practical in actual application [5,6]. Liu et al. [2] studied the dispersion relationship of a three-dimensional periodic structure consisting of cubic arrays of coated lead spheres immersed in an epoxy matrix, in which the first Local-resonant band gap was observed. Interestingly, it was found that the Local-resonant band gaps can exist in a frequency ranges of two orders of magnitude lower than the one resulting from the Bragg scattering mechanism, which means the size of the LRPS could be designed smaller. Since then, extensive work has been reported on dispersion properties of the Local-resonant periodic structures (LRPSs) [7]. More recently, based on the concept of Local-resonant, *elastic metamaterials* were proposed and have gained much attention due to their unique microstructure designs to achieve effective dynamic material properties which cannot be observed in nature [8–10].

Through it has been established in the field of solid-state-physics for many years, the periodic structure theory was introduced into the field of civil engineering only recently. We think that introducing the periodic structure theory into civil engineering has two interesting topics that attract people's attention. First, as using the filtering effect for elastic waves, novel multi-functional materials/structures could be designed for modern buildings. For example, inspired by the concept of the frequency band gap, Shi and his co-workers [11,12] proposed the

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so-called periodic foundation for civil engineering structures to isolate the harmful seismic wave. In using this method, the traditional solid foundation will be replaced by the periodic foundation. When the pattern of the earthquake event energy reaches the periodic foundation, a large amount of seismic energy will be prohibited and seismic responses of the upper structure will be attenuated. Through theoretical, numerical and experimental studies, the feasibility of the proposed periodic foundation have been proved [13–16]. Integrating critical concepts from several seemingly disparate fields including *plasmonics*, *metamaterials*, *elasticity*, and *geophysics*, Colombi et al. [17,18] designed a seismic *metawedge* capable of creating a seismic rainbow to convert the destructive Rayleigh seismic waves into the harmless bulk shear wave. In a similar way, burying sub-wavelength resonant structures under the soil surface, Palermo et al. [19,20] proposed a seismic *metabarrier*. In that work, a scaled experimental model was fabricated and tested to prove the shielding performance of the proposed *metabarrier*. It was demonstrated that surface ground motion can be reduced by up to 50% in frequency region below 10 Hz. Incorporating the coated lead balls into a short fiber reinforced cementitious composite, Li et al. [21,22] developed a new concrete. Their experimental results showed that the new concrete has much better sound proofing capability and has good potential in application in low frequency sound shielding. Inspired by the *metamaterials* used for the manipulation of electromagnetic and acoustic waves, Mitchell et al. [23] introduced a concept for a new type of concrete, named *metaconcrete*, capable of reducing the energy transmitted by dynamical loading without the use of dissipative element. Their further work found that the incorporation of *metaconcrete* aggregates into a slab design can modify the behavior of a mortar slab, reducing the effect of a blast loading when the slab is also undergoing erosion and fracture [24]. Cheng and Shi [7,13] investigated the frequency band gaps of two-dimensional periodic rubber concrete panels. Their numerical results showed that vibration can be reduced significantly by using a periodic rubber-concrete panel with only three units. Utilizing interactions from the resonant motions and the viscoelastic effects of the constitutive material, Chen et al. [25] presented an elastic metamaterial with multiple dissipative resonators for broadband wave mitigation. The results of this study showed that the proposed elastic metamaterial could be used in developing new multifunctional composite materials to suppress the shocks or blast waves which may cause severe local damage to engineering structures. Based on the concept of metamaterials, Maleki and Khodakarami [26] proposed a kind of seismic cloaking metamaterial, termed MetaSoil (MS), to reduce the seismic amplification on hills subjected to in-plane waves. Adopting both the Bloch's theory and the classical vibration analysis, Chatzi and her co-workers [27,28] studied the feasibility on the attenuation of strong ground motions using finite periodic lattices of mass-in-mass barriers.

Second, the periodic structure theory offers a new approach to analyze the dynamic properties of civil engineering structures. It is known that, for the sake of convenience, many civil engineering structures are built in a periodic manner, like high-rise buildings, continuous multi-span bridges, orthotropic steel bridge decks, railway track structures, multi-row piles, and so on. Despite being finite in reality, these structures can be analyzed as infinite sequences of identical elements connected to each other (‘periodic structures’). Recently, using the tools of spectral analysis of Bloch waves in a periodic system, Brun et al. [29,30] addressed a mathematical model describing the dynamic response of an elongated bridge (the S’Adde Bridge in Sardinia, Italy) supported by elastic pillars. Their theoretical results showed that the periodic structure theory is an appropriate approach to evaluate the frequency intervals for a finite elongated bridge. On the basis of these works, Carta et al. [31,32] studied the dynamic flexural behavior of a long bridge (the ‘Brabau’ bridge across the river Tirso in Italy), modelled as an infinite periodic structure, in which the contributions of the bridge's structural elements on the dispersive properties of the bridge were investigated in detail. It was proved that the proposed approach can be implemented as a simple procedure to design structures with repetitive units, with the advantage of simplifying numerical simulations and reducing the computational cost. Based on the periodic structure theory, Thompson [33] obtained the dispersion relationship and the receptances of an infinite free rail. In that work, taking advantage of the fact that the cross-section remains constant along the rail and that its length is infinite, the structure was considered as a periodic structure with arbitrary period of 10 mm. It was found that the results of the model compare favorably with measurements taken from literature. Based on the band-gap theory, Wang et al. [34] studied the band-gap behaviors and formation mechanisms of periodic track structures. It was found that formulation mechanisms of band-gaps in periodic track structures can be explained by the Bragg scattering mechanism and the Local resonance mechanism. Extending the multi-row pile structure into the infinite periodic pile-soil structure, Huang and Shi [35,36] studied the filtering effect of the periodic pile barriers for elastic vibrations induced by traffic loads. Their numerical results showed that vibrations with frequencies in SBs can be reduced significantly. Introducing the so-called ‘open’-type periodic unit, Lu and Jeng [37] conducted a numerical study to analyze the dispersion property of a viaduct possessing periodic characteristics undergoing out-of-plane vibration. In another contribution, considering the theoretical infinite periodic viaduct mode, the resonance and cancellation phenomena occurring in the pile-supported periodic viaduct subjected to multiple equidistant moving loads were investigated [38]. Proposing a second-order computational homogenization method, Bacigalupo and Gambarotta [39,40] derived the overall constitutive moduli and inertia properties, and evaluated the influences of the materials characteristic lengths on the structural response and the propagation of elastic waves in the periodic masonry structure. Through the Floquet-Bloch approach, Bacigalupo and Gambarotta [41] investigated the dispersive waves in two-dimensional blocky materials with periodic microstructure made up of equal rigid units, having polygonal centro-symmetric shape with mass and gyroscopic inertia, connected with each other through homogeneous linear interfaces.

In this work, introducing the periodic structure theory of in the field of solid-state-physics into the field of civil engineering, dynamic dispersion properties of multi-story frame building structures are analyzed. As we know, multi-story frame building structures are widely used in civil engineering due to their simple beam-column load-transmission system. Geometrically, multi-story frame building structures could be viewed as a periodic system with its standard floor arranged in vertical direction periodically. Therefore, in analogy to the *phononic crystals* and the *elastic metamaterials*, the periodicity arrangement endows multi-story frame building structures some interesting dispersion properties. However, to the best of our knowledge, until now little research has been conducted on this topic. Here, corresponding to the dynamic properties of building structures in the two horizontal directions, dispersion properties of flexural wave propagating in the vertical direction in a periodic frame structure are considered. The main novelties of this work can be attributed to the following points: At first, in order to perform the Bloch analysis, the theoretical infinite periodic model for multi-story frame structures is developed for the first time. And, to avoid the complex theoretical derivations of the traditional transfer matrix method (TMM), the state-space-transfer-matrix method (SSTMM) is developed to solve the dispersion equation. Second, a comprehensive parametric study is conducted to investigate the influences of the typical frame structure parameters (including the geometrical and physical parameters) on the dispersion properties of the periodic frame building structures. Third, dynamic responses of a practical 6-story reinforced concrete frame building are investigated through harmonic analysis and time history analysis, from which the filtering effect of the periodic frame building structure is validated. Interestingly, it is found that seismic responses of the periodic frame structure are very large when the main frequency region of the ground motion falls in the PBs of the upper structure; however, seismic