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The effect of sliding on the rocking instability of multi-rigid block assemblies under ground motion

Anthony N. Kounadis

The Academy of Athens, Athens, Greece

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ABSTRACT

The highly nonlinear differential equations describing the complex rocking-sliding instability of a freely standing on the ground multi-rigid block assembly under horizontal ground motion are analytically derived using an energy variational approach. A major step for deriving these equations is the evaluation of the displacements (horizontally and vertically) of the gravity centers of all rigid blocks of the assembly, after rocking initiation of the lower block and separation of all blocks to each other. This is conveniently achieved by combining the corresponding displacements of the same assembly without sliding (based on previous author's analyses) with the additional displacements due to slidings between consecutive blocks except the lower one with the ground. The increase of the magnitude of sliding along the height of the assembly (from bottom to the top) implies respective increase in the loss of energy. Thus, the beneficial effect of sliding on the minimum amplitude ground acceleration (leading to overturning) becomes more pronounced as the number of rigid blocks of the assembly increases. It was shown qualitatively and quantitatively that a very small sliding in a two-rigid block assembly may provoke a significant increase of the minimum amplitude ground acceleration (stabilizing considerably the assembly). It was also proved via a qualitative analysis that the number of configuration patterns to be examined can be substantially reduced which confines considerably the computational effort. Moreover, some new findings for the rocking-sliding response for one- rigid block systems are also presented contributing to the rocking-sliding analysis of multi-rigid block assemblies.

1. Introduction

The overturning response under ground motion of ancient (Hellenic) multispondyle columns (with spondyles freely supported one to another) carrying atop free- standing statues has been a research topic of increasing interest over the last decades. This is particularly due to the excellent dynamic stability features thanks to which these structural systems (multispondyle column-statue) have survived strong earthquakes in the past. A further advantage of the system is that the presence of the large statue atop the column increases the natural period and subsequently the earthquake resistance of the system in connection with the acceleration spectra of Eurocode 8 (Kounadis [1]). The stability *superiority* (under ground excitation) of ancient multi-spondyle columns compared *to monolithic* ones was early recognized in other places of the world (Italy, Middle East, etc) in which similar structural systems have been constructed. A typical example is the two columns (with thirteen spondyles) in the front of the neoclassical building of the Academy of Athens carrying atop the free- standing large statues of Athena and Apollo constructed in 1885 [2].

An exact dynamic analysis of such complex structural systems (governed by a large number of highly nonlinear differential equations) as shown by the author [2] is *impossible* and only *approximate* but *reliable* rocking instability analyses (supported by experimental data referring to the total loss of energy) can be applied. The total loss of energy, due to the combined effect of *impact* and (very *small) sliding*, between spondyles, varying along the height of the multispondyle column (becoming maximum at its top) implies a *significant reduction* in the magnitude of ground excitation at the base of the statue. Thus, the remaining amount of ground excitation is, in general, insufficient to overturn the statue [3]. Such a combined effect of energy dissipation (of the multispondyle column) can be evaluated only experimentally.

Reviewing the present state of the art **all** existing analyses dealing with the rocking response of one rigid block system including very few studies of two and only one of three rigid blocks (Kounadis and Papadopoulos [4]) are based on the assumption that the **friction** between the rigid block and the ground as well as between consecutive blocks is *sufficiently large* to **exclude** sliding. To the best knowledge of the author the *first* study dealing with the numerical evaluation of the *sliding effect* on the rocking response of **one**-rigid block systems was recently published (Kounadis [5]).

E-mail address: kounadis@bioacademy.gr.

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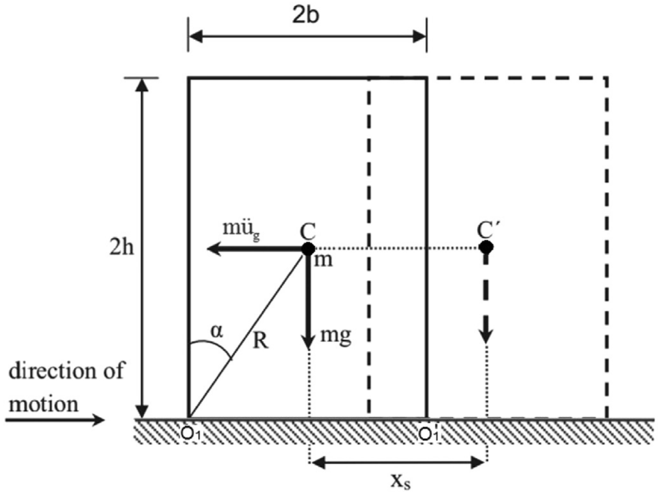

Fig. 1. Free-standing rectangular rigid block under its own weight mg subject to sliding x_s due to horizontal ground motion.

In view of the importance of sliding effect (on the minimum amplitude ground acceleration), the purpose of this work is the extension of this effect, valid for one block, to assemblies with more than two rigid blocks. To this end, it is more convenient to revisit the *single* block rocking study mentioned above [5] using a simpler methodology based on a *qualitative* analysis which facilitates the subsequent analysis of multi-rigid block assemblies. In the framework of this analysis some byproducts are obtained which clarify or verify previous author's findings that contribute to the rocking-sliding analysis of multi-rigid block assemblies. A major step for the last complex instability analysis is the derivation of the highly nonlinear differential equations of rocking-sliding motion, which is achieved through a sophisticate and comprehensive methodology.

2. Problem description

Let us first consider *the effect of sliding* in the simple case of a rectangular rigid block with dimensions $2b \times 2h$ and mass m (Fig. 1), being in vertical equilibrium under its own weight mg, where g is the gravity acceleration. Rigid block under ground motion may translate with the ground, slide, rock or slide-rock (Shenton [6]). As very recently shown [5], this action depends on the angle $\alpha = \tan^{-1}(b/h)$, on the roughness of the surfaces between the block and the ground (expressed through the friction coefficient μ) as well as on the form, magnitude and duration of ground excitation. Assuming dry friction Coulomb laws, the friction (horizontal) force F (before to rocking initiation) is proportional to the normal (vertical) force mg multiplied by the kinetic coefficient of friction μ (< 1), being always smaller its static value, i.e. $F = \mu mg$. Shortly after the initiation of rocking the mass center of the block C (due to rotation) is going up causing an opposite (vertical) inertia force N_i. Hence, the total normal (vertical) force is $N_T = mg + N_i$. The total friction (horizontal) force $F_t = \mu(mg + N_i)$ takes its *maximum* shortly after rocking initiation (when the block ceases to be in contact with the ground) and then decreases as the angle of the block rotation $\theta(t)$ increases (absolutely). This is due to the fact that the *kinetic friction coefficient* μ varies during motion, increasing gradually from the onset of ground excitation (and during the time the block is entirely in contact with the ground) until shortly after rocking initiation, occurring at a time $t = t_0$ at which μ becomes maximum, $\mu = \mu_{\max}$. For the interval $t > t_0$ the friction effect of inertia forces may be *ignored* as being of minor importance.

Given that the *friction force* is always *opposite* to the direction of the ground motion the differential equation of free motion governing the *sliding* x_s (under horizontal ground motion) is

$$\ddot{x}_s = \mu N_T \text{sgn}[\dot{x}_s] = \mu(mg + N_i)\text{sgn}[\dot{x}_s] \quad (1)$$

which ignoring N_i (acting after rocking initiation) becomes $\ddot{x}_s = \mu g\,\text{sgn}[\dot{x}_s]$ (Younis and Tadjbakhsh [7], Taniguchi [8]).

An *approximate* but *reliable* estimation of the fluctuation of the kinetic friction coefficient μ for a given rigid block and ground excitation is obtained through the condition of excluding sliding. Namely, the horizontal reaction $H(t)$ at the point O_1 due to the ground excitation, if sliding is excluded, is smaller or equal than the friction force $\mu V(t)$, where $V(t)$ is the vertical reaction at point O_1 (Shenton [6]), i.e.

$$|H(t)/V(t)| \le \mu \quad (2)$$

As stated previously the above ratio of forces at $t = t_0$ becomes maximum, and thus also $\mu = \mu_{\max}$. Such a *maximum* can also be obtained through the necessary condition [5].

$$|\dot{H}V - \dot{V}H| = 0 \quad (3)$$

The *minimum* value of the above ratio of forces and $\mu_{\min}$ are obtained at time $t = 0$ as follows [5].

$$|H(0)/V(0)| = \tan\alpha < \mu_{\min} \quad (4)$$

The analytic expressions of $H(t)$ and $V(t)$ are also given in [5].

2.1. Rocking initiation without sliding

A remarkable observation based on a qualitative analysis is related to the case of **sufficiently large friction** which **excludes sliding** (i.e. $x_s = 0$). In this case the direction of the ground (horizontal) acceleration from the left to the right (Fig. 1) gives rise to the development of inertia forces which after counterbalance the gravity force mg provoke rocking initiation. Assuming *trivial* initial conditions $\theta(0) = \dot{\theta}(0) = 0$ *rocking initiation* may occur when, at $t = 0$, the bending moment of the inertia force overcomes the bending moment of the gravity force, i.e.

$$m\ddot{u}_g(0)R\cos\alpha \ge mgR\sin\alpha$$

or

$$\ddot{u}_g(0) \ge g\tan\alpha \quad (5)$$

where the equality assures the condition for sliding.

The following question now arises: for the (positive) direction of ground horizontal motion (Fig. 1) *rocking initiation* will occur via a *negative* (anticlockwise) or via a *positive* (clockwise) rotation $\theta(t)$, i.e. via $\theta(t) < 0$ about pivot point O or $\theta(t) > 0$ about pivot point O′?

Clearly, relation (5) assuring rocking initiation (without sliding) is satisfied a fortiori if the translational (horizontal) inertia force were applied above C as shown in Fig. 2a at a point C′. This situation is *equivalent* to that shown in Fig. 2b, where the force $m\ddot{u}_g(0)$ and its moment about C, being the *rotational* inertia of the block $J_c\ddot{\theta}(0)$ (J_c is the block polar moment of inertia about C) are applied at C. Both these inertia quantities have the *same* (anticlockwise) direction.

The important **conclusion** which is drawn is: for a *positive* external ground acceleration both these inertia (translational and rotational) quantities, being of the **same** (anticlockwise) direction will apparently induce rocking *initiation* through a *negative* rotation $\theta(t) < 0$. Hence, rocking initiation through $\theta(t) > 0$ (which may lead to overturning) is **impossible**. The existing relevant overturning analyses for $\theta(t) > 0$ based on the old landmark work by Housner [9] were unnecessary as meaningless. Using such a rocking overturning analysis under one wave (sine and cosine) ground pulse Kounadis [10] showed that rocking initiation with $\theta(t) > 0$ implies a physically **unaccepted** solution which is rejected. However, rocking initiation with $\theta(t) > 0$ (which may lead to overturning) may occur in case of suitable *nontrivial* initial conditions, e.g. for $\theta(0) \neq 0$ or $\dot{\theta}(0) \neq 0$ (Kounadis [10,11]). Another finding is that rocking initiation under *positive* ground acceleration may occur for $\theta(t) > 0$ (leading possibly to overturning) in case of **partial friction** which allows *small sliding* x_s implying the development of the force $m\ddot{x}_s$, being *opposite* to the inertia quantities. This is discussed below.