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# Investigation of ground vibrations induced by trains moving on saturated transversely isotropic ground

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A B S T R A C T

A 2.5D FEM (finite element method) is used to investigate the effects of soil parameters of transversely isotropic (cross anisotropic) saturated soil on ground vibrations and excess pore water pressures induced by moving train loads. The governing equations of transversely isotropic saturated soil are derived from the Boit's theory in frequency domain by applying the Fourier transform with respect to time, and 2.5D FE model is then established using Galerkin method. Correctness of the proposed model is validated with published data. Numerical results illustrate that the decrement of vibration amplitude and excess pore water pressure caused by the increment of vertical elastic modulus is more significant than that of the horizontal direction. Poisson ratios in both directions have little effect on ground vibrations, while an increase in horizontal Poisson ratio results in a significant increment in excess pore water pressure.

## 1. Introduction

Railway train has been a major mode of public transportation, especially in China. With the rapid development of high-speed railways, the environmental vibration caused by moving trains is becoming more widely concerned. Natural soils widely distributed in coastal area usually is exhibited the characteristic of cross-anisotropy or transverse isotropy due to sedimentation or consolidation. Therefore, researchers should pay more attention on the vibrations of transversely isotropic (cross anisotropic) ground.

Many experimental methods are adopted to study the property of transversely isotropic soil. For example, Kuwano et al. [1] used bender elements and trigger-accelerometers to measure elastic wave velocities transmitted vertically in triaxial specimens of sand, gravel and glass beads. Nishimura [2] adopted high-precision triaxial apparatus to study cross-anisotropic deformation characteristics of natural sedimentary clays. Other researchers studied the analytical solution of wave propagation in transversely isotropic ground. Papargyri-Beskou et al. [3] studied the wave propagation in gradient elastic solids and structures. Zymnis et al. [4] presented closed-form analytical solution for estimating far-field ground deformations caused by shallow tunneling in a linear elastic soil mass with cross-anisotropic stiffness properties. Ahmadi and Eskandari [5] analyzed the vibrations of rigid circular disk and strip embedded in a transversely isotropic solid. Ogden and Singh [6] investigated the effect of rotation and initial stress on the propagation of waves. Recently, Ai and Ren [7] analyzed the vibration of a transversely isotropic solid subject to a moving loading using the analytical element method.

Apart from the experimental and analytical studies, numerical method is becoming a promising method in study of this problem with its feasibility for dealing with actual problems with irregular geometry. Abedrrahim [8] presented a coupling method of finite and hierarchical infinite elements to solve a non-homogeneous cross-anisotropic half-space subjected to a non-uniform circular loading. These methods showed good performance in predicting vibration in non-homogenous soils, however, such models are rather expensive in calculation time and memory space. To improve the computational efficiency and ensure the accuracy of computational model, a 2.5D FEM was used for solving the ground vibrations induced by a moving train [9–12]. The 2.5D FEM conducts Fourier transform along the train moving direction, and solves the three dimension problem with a two dimensional FE grids which is dispersed on section perpendicular to the train moving direction. It is firstly used in seismic analysis, and then employed to solve dynamic response under train loads by Yang and his collaborators [9,10]. Nevertheless, published results using 2.5 D FEM are all in homogenous or layered elastic and saturated soils, study on ground

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borne vibration under moving train loads in transversely isotropic saturated soil is not yet reported.

In view of this, based on Biot theory and the Galerkin method, this paper establishes a 2.5D FEM of transversely isotropic saturated soil together with flow viscoelastic boundary conditions, to predict ground vibrations in such soils subjected to train loads; and the effects of mechanical parameters of transversely isotropic saturated soil on the ground vibration and excess pore water pressure are studied in detail.

## 2. Equations of *u-p* format for 2.5D FEM

The finite element model is the same as that in Ref. [11], track and ground are simplified as Euler-Bernoulli beam and transversely isotropic saturated porous medium, respectively. The train moves along the track with a velocity $c$, the expression of train loads can be seen in Ref. [11]. The material and geometric properties are assumed to be constant along the train moving direction. Coordinates system of the finite element model is the same as that in Ref. [11], where $x$ is the train moving direction, $y$ is the direction perpendicular to track, and $z$ is the vertical direction, the track center is the origin of coordinates. In addition, the height of embankment is set to be 1.0 m, and underground water level is at the ground surface.

According to Biot's theory of wave propagation in fluid-saturated porous medium, the dynamic motion equations of a fully saturated poroelastic medium can be expressed as follows [11]:

$$\sigma_{ij,j} + F_i = \rho u''_i + \rho_f W''_i \tag{1}$$

$$-\frac{n}{K_f}p' = W'_{i,i} + u'_{i,i} \tag{2}$$

$$-p_{,i} = \rho_f u'' + \frac{\rho_f}{n}W''_i + \frac{\rho_f g}{K_d}W'_i \tag{3}$$

in which $\sigma_{ij}$ is the stress of porous medium and $F_i$ is the body force of the solid skeleton; $\rho$ and $\rho_f$ denote the bulk density of the porous medium and the density of the pore fluid, $\rho = \rho_s(1-n) + n\rho_f$, in which $\rho_s$ is the density of the solid skeleton and $n$ is the porosity of the porous medium; $W_i = n(w_i - u_i)$ is the average displacement of the pore fluid relative to the solid skeleton, in which $w_i$ and $u_i$ denote the infiltration displacements of pore fluid and the average displacement of solid skeleton, respectively; $p$ is the excess pore water pressure and $g$ is the acceleration of gravity; $K_f$ and $K_d$ are the bulk modulus of pore fluid and the permeability of the porous medium, respectively; (′) indicates differentiation with respect to time $t$.

The Fourier transformation of function $u(x, y, z, t)$ with respect to $x$-coordinate and time $t$ is defined by:

$$\tilde{\bar{u}}(\varepsilon_x, y, z, \omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u(x, y, z, t)\mathrm{e}^{\mathrm{i}\varepsilon_x x}e^{-\mathrm{i}\omega t}\mathrm{d}x\mathrm{d}t \tag{4}$$

The corresponding inverse transforms with respect to $\varepsilon_x$ and $\omega$ is given by:

$$u(x, y, z, t) = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \tilde{\bar{u}}(\varepsilon_x, y, z, \omega)\mathrm{e}^{-\mathrm{i}\varepsilon_x x}\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}\varepsilon_x\mathrm{d}\omega \tag{5}$$

where $\omega$ and $\varepsilon_x$ represent circular frequency and the horizontal wavenumber corresponding to $x$-direction, respectively.

Based on the generalized Hooke's law, stress-strain relationship and effective stress principle of soil, the relationship between stresses and displacements of soil are given as:

$$\begin{cases} \sigma_x = C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y} + C_{13}\frac{\partial w}{\partial z} - p & \tau_{yz} = C_{44}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \sigma_y = C_{12}\frac{\partial u}{\partial x} + C_{11}\frac{\partial v}{\partial y} + C_{13}\frac{\partial w}{\partial z} - p & \tau_{zx} = C_{44}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \\ \sigma_z = C_{13}\frac{\partial u}{\partial x} + C_{13}\frac{\partial v}{\partial y} + C_{33}\frac{\partial w}{\partial z} - p & \tau_{xy} = C_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \end{cases} \tag{6}$$

where $u$, $v$ and $w$ are respectively displacements of soil skeleton in $x$, $y$ and $z$ directions; $C_{ij}(i, j = 1, 2, 3, 4, 6)$ are mechanical parameters of transversely isotropic soil, which can be expressed in horizontal and vertical elastic moduli, horizontal and vertical Poisson's ratio and the shear modulus. Elastic modulus in complex form is introduced to account for the material damping.

In order to eliminate time derivatives in Eq. (3), the Fourier transformation with respect to time is performed on Eq. (3). As a result, the equation is transformed into the frequency domain. By using the derivative nature of Fourier transform, the following equation can be obtained:

$$W_i = F(\omega^2\rho_f\bar{u}_i - \bar{p}_i) \tag{7}$$

in which $F = nK_d/(\mathrm{i}\omega\rho_f gn - \omega^2K_d\rho_f)$, variables with a bar above indicate the components in frequency domain.

Substituting Eqs. (6) and (7) into Eq. (1), and then performing Fourier transformation with respect to time, the balance equations of mechanics parameters in frequency domain are given by:

$$\begin{cases} (C_{11}\frac{\partial^2}{\partial x^2} + C_{66}\frac{\partial^2}{\partial y^2} + C_{44}\frac{\partial^2}{\partial z^2})\bar{u} + (C_{12} + C_{66})\frac{\partial^2\bar{v}}{\partial x\partial y} + \\ (C_{13} + C_{44})\frac{\partial^2\bar{w}}{\partial x\partial z} - \bar{P}_{,x} + \omega^2\rho\bar{u} + \omega^2\rho_f F(\omega^2\rho_f\bar{u} - \bar{P}_{,x}) = 0 \\ (C_{12} + C_{66})\frac{\partial^2\bar{u}}{\partial x\partial y} + (C_{66}\frac{\partial^2}{\partial x^2} + C_{11}\frac{\partial^2}{\partial y^2} + C_{44}\frac{\partial^2}{\partial z^2})\bar{v} + \\ (C_{13} + C_{44})\frac{\partial^2\bar{w}}{\partial y\partial z} - \bar{P}_{,y} + \omega^2\rho\bar{v} + \omega^2\rho_f F(\omega^2\rho_f\bar{v} - \bar{P}_{,y}) = 0 \\ (C_{13} + C_{44})\frac{\partial^2}{\partial y\partial z}\bar{v} + (C_{44}\frac{\partial^2}{\partial x^2} + C_{44}\frac{\partial^2}{\partial y^2} + C_{33}\frac{\partial^2}{\partial z^2})\bar{w} + \\ C_{44} + C_{13})\frac{\partial^2\bar{u}}{\partial x\partial z} - \bar{P}_{,z} + \omega^2\rho\bar{w} + \omega^2\rho_f F(\omega^2\rho_f\bar{w} - \bar{P}_{,z}) = 0 \end{cases} \tag{8}$$

Similarly, Fourier transformation with respect to time is performed on Eq. (2). Then by substituting the results obtained into Eq. (7), the balance equation of fluid in frequency domain is expressed as:

$$(F\omega^2\rho_f K_d + K_d)(\frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{v}}{\partial y} + \frac{\partial\bar{w}}{\partial z}) + n\bar{P} - FK_d(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})\bar{P} = 0 \tag{9}$$

Stress boundary conditions and flow boundary condition in frequency domain of the FEM model are given as:

$$\begin{cases} \sigma_x l + \tau_{xy}m + \tau_{xz}n = f_x \\ \tau_{yx}l + \sigma_y m + \tau_{yz}n = f_y \\ \tau_{xz}l + \tau_{zy}n + \sigma_z m = f_z \end{cases} \tag{10}$$

$$\bar{q} = \rho_f g\bar{v}_n = -K_d(\frac{\partial}{\partial x}l + \frac{\partial}{\partial y}m + \frac{\partial}{\partial z}n)\bar{P} \tag{11}$$

where $f_i(i = x, y, z)$ are components of external forces in $x$, $y$, $z$ directions; $l$, $m$, $n$ are directions cosine, respectively; $\bar{q}$ is the flow of pore water; $\bar{v}_n$ is flow velocity of pore water.

Combining the constitutive equation and applying the Galerkin method to Eqs. (8)–(11), and then incorporating the developed shape function and performing wave-number expansion on the resulting equation in $x$-direction, the 2.5D FEM governing equations in wavenumber domain and frequency domain can be derived by conventional finite element method, which are given by:

$$(\boldsymbol{K_{up}} - \boldsymbol{M_{up}})\tilde{\bar{\boldsymbol{u}}} + (\boldsymbol{Q'_{up}} - \boldsymbol{Q_{up}})\tilde{\bar{\boldsymbol{P}}} = \tilde{\bar{\boldsymbol{f}}}^{s}_{up} \tag{12a}$$

$$(\boldsymbol{H_{up}} + \boldsymbol{S_{up}})\tilde{\bar{\boldsymbol{P}}} + \boldsymbol{Q''_{up}} = \tilde{\bar{\boldsymbol{f}}}^{q}_{up} \tag{12b}$$

where $\boldsymbol{K_{up}}$ is stiffness matrix; $\boldsymbol{M_{up}}$ is mass matrix; $\boldsymbol{Q''_{up}}$, $\boldsymbol{Q'_{up}}$ and $\boldsymbol{Q_{up}}$ are solid and fluid coupling matrixes; $\boldsymbol{H_{up}}$ and $\boldsymbol{S_{up}}$ are Jacobian matrixes; $\bar{\mathbf{f}}^{s}_{up}$ and $\mathbf{f}^{s}_{up}$ are equivalent node load vectors; $\bar{\mathbf{u}}$ is node displacement matrix; variables with '∼' above indicate the component in wavenumber domain.

Artificial boundary has a non-negligible influence on the calculation accuracy. Referring to Gao et al. [11], this paper adopted a 2.5D viscoelastic dynamic artificial boundary to model the wave propagation in