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Interpretation of the velocity measured in buildings by seismic interferometry based on Timoshenko beam theory under weak and moderate motion

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ABSTRACT

Application of seismic interferometry in buildings gained interest in the recent years for structural health monitoring. It allows us to derive the shear wave velocity for an equivalent homogeneous medium representing the structure. Previous authors suggested using a shear beam model to compute the fundamental frequency of the structure out of this velocity. This model is however not adapted to a large part of existing buildings having different behaviors. In this paper, we propose a correction factor from shear beam and based on the Timoshenko beam to link the fundamental frequencies with the observed pulse velocity and the relative effects of shear and bending. This factor provides corrections up to 60% in frequency with respect to the shear beam model. The proposed correction factor shows that the higher velocities observed in the literature for shear wall buildings compared to frame buildings is compensated by their bending flexibility, resulting in resonance frequencies scaling similarly with building height. This model is applied to an 18-story reinforced concrete shear wall building (Ophite tower). The observed pulse velocity obtained by seismic interferometry in this building was about 500 m/s correlated to the resonance frequency by the correction factor. We show that variations of velocity in this structure and the well-studied Factor building (California), related to non-linear behavior at low-strains (down to 10^{-5}), can be retrieved with seismic interferometry, demonstrating that this method is sensitive enough for structural health monitoring.

1. Introduction

Since Snieder and Safak [1] proposed an application to buildings of what they called “seismic interferometry by deconvolution”, extensive research related to this topic in the fields of earthquake engineering and engineering seismology has been reported [2–14]. In this method, continuum mechanics models such as the shear beam model have been effectively used to study wave propagation and resonant frequencies and modes in buildings. As suggested by previous authors [15,16], the impulse response function (IRF) of the vertical structure is computed by modeling wave propagation in a continuous linear system by correlation among the horizontal motion observed at different elevations in the building. The IRF reflects the structural properties like damping and stiffness. Therefore, other authors exploited this method, also called Impulse Response Analysis, for Structural Health monitoring applications, i.e. to detect and localize damage in the structure through the computation of the variation of the obtained wave propagation velocity, using earthquake or ambient vibrations [11,12,17–21]. Snieder and Safak [1] and later Nakata et al. [9] have shown that according to the principle of the deconvolution, the IRF is independent from soil-structure interaction SSI. Through a structural model based on a shear beam including SSI, Todorovska [22] demonstrated also that the fixed-base frequency (i.e. without coupling with the soil) was related to the wave travel time from the base to the top, unaffected by SSI considering a broadband IRF, as confirmed in Rahmani et al. [23].

Michel et al. [7] showed that in presence of shear walls, the proxy based on the wave travel time (obtained using the fixed-base IRF by deconvolution) and the shear beam model was systematically over-estimating the resonance frequency. Their interpretation at that time was the presence of a shear-wave velocity gradient in the structure. Ignoring the bending motion in interferometric interpretation produces artificial softening in the IRF [4] that may be misinterpreted as SSI.

As suggested by Boutin et al. [24], the shear-beam model is more appropriate for moment resisting frame structures while the bending beam model corresponds to structures designed with shear walls. Boutin et al. [24] proposed a continuous Timoshenko beam model to

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characterize the response of vertical fixed-based structures. As for the Euler-Bernoulli beam model, the Timoshenko beam model assumes that plane sections remain plane. However, in the Timoshenko beam model the sections do not necessarily remain perpendicular to the neutral axis as in the Euler-Bernoulli beam to account for shear and rotary effects. Boutin et al. [24] provided then a simple but physics-based relationship to compute the shear-to-bending stiffness ratio using the ratio between higher-mode resonance frequencies and the fundamental resonance frequency. Fitting a layered Timoshenko beam model, Ebrahimian and Todorovska [25] interpreted the IRF obtained by deconvolution accounting for the bending behavior of structures. Moreover, they showed that the wave train obtained by deconvolution was dispersive in presence of bending motion, as suggested by the Euler-Bernoulli equation. Consequently, considering a shear beam or an Euler-Bernoulli beam model to interpret IRF by deconvolution provides, respectively, an under- or over-estimated phase velocity at higher wave number, while the Timoshenko beam model provides a more accurate wave solution even in the high frequency range. The Timoshenko beam model is then more appropriate to interpret the pulse wave velocity in terms of frequency response.

The aim of this paper is to propose a simple and physics-based relationship between shear wave velocity and resonance frequencies of buildings, using the analytical formulation of the simplified Timoshenko beam model provided by Boutin et al. [24]. The paper first presents the Timoshenko beam (TB) as proposed by Boutin et al. [24] compared to the one of Ebrahimian and Todorovska [5]. The dispersion relationship is derived, as well as the simple relationship giving the resonance frequencies. The case-study of a fixed-based reinforced concrete (RC) shear walls instrumented building in France (Ophite Tower – OT) illustrates the approach. The interferometry technique is then applied to earthquake recordings in the OT to ultimately retrieve the shear wave velocity and is compared to velocities published in literature. The variations of this velocity with respect to frequency are investigated as well as its variations with the amplitude of seismic loading and compared to that observed at the Factor building, located in the UCLA campus (California).

2. Timoshenko beam model

In the shear beam model used by Snieder and Safak [1] and many other studies, the bending stiffness, represented by the parameter EI (E Young's modulus, I moment of inertia), is not considered because it is much larger than the shear stiffness, i.e. the contribution of the bending moment to the building displacement is neglected. This assumption has been validated for Moment Resisting Frame (MRF) buildings such as the Factor building [6,12]. In these structures, the rigid connections between the floors and the structure make the inertia I of the structural elements large enough to minimize the rotation due to bending. Conversely, the thin vertical elements provide a relatively low shear stiffness that allows the majority of the displacement to occur in shear. However, this is no more the case for structures with continuous shear walls. In these cases, the length of the walls imposes a much larger shear stiffness that competes with the bending stiffness inducing a coupling between shear and bending effects.

In order to model regular existing buildings using experimental data, Boutin et al. [24] proposed a model beam accounting for shear and bending based on the Timoshenko beam theory, and neglecting the rotation inertia, later referred as BO05. Ebrahimian and Todorovska [5] proposed also a detailed formulation, referred here as ET14, with illustration to wave propagation in building. In addition to the bending stiffness EI, shear stiffness $K = Gk_GA$ is also introduced in the Euler-Bernoulli beam equation [24], with A the section of the beam, G the shear modulus of the equivalent medium, i.e. including structure and voids, and k_G the shear adjustment factor depending on the shape of the cross section of the beam and reflecting the non-uniform distribution of shear stress and shear strain over the section [26]. In harmonic regime, the governing equation of the horizontal translation motion of the beam $U(z, t) = e^{-i\omega t}U(z)$ is given as follows (model BO05):

$$EIU^{(4)}(z) + \frac{EI}{K}\rho A\omega^2 U^{(2)}(z) = \rho A\omega^2 U(z) \tag{1}$$

with the upper script () indicating the order of the time derivative, t the time, z the position along the beam, ρ the density of the equivalent medium and ω the angular frequency.

In order to evaluate the dominant mode of behavior, Boutin et al. [24] introduced a dimensionless parameter C based on Jensen [27]:

$$C = \frac{EI}{K\left(\frac{2H}{\pi}\right)^2} \tag{2}$$

with $2H/\pi$ (H the length of the beam) characterizing the dispersive nature of the beam. In Eq. (1), the Timoshenko beam degenerates into a Euler-Bernoulli (bending) beam when C tends to 0 and into a pure shear beam when C tends to infinity.

Whatever the damping, equations of Euler-Bernoulli and Timoshenko beams are dispersive, i.e. a propagating wave exists in the equivalent-beam building, its velocity depending on the frequency. The dispersion is expressed by assuming the solution of Eq. (1) of the form $U(z) = U_0e^{ikz}$ with k the wavenumber of the corresponding plane wave. From Eq. (1), the dispersion relationship is therefore given by:

$$EIk^4 - \frac{EI}{K}\rho A\omega^2 k^2 - \rho A\omega^2 = 0 \tag{3}$$

and the solutions of this equation are $k_{1,2}$:

$$k_{1,2} = \sqrt{\frac{\frac{EI}{K}\rho A\omega^2 \pm \sqrt{\left(\frac{EI}{K}\rho A\omega^2\right)^2 + 4EI\rho A\omega^2}}{2EI}} \tag{4}$$

Considering the expressions of the shear wave velocity $c_s = \sqrt{\frac{G}{\rho}}$ and dimensionless parameter C (Eq. (2)), the solutions can be rewritten as:

$$k_{1,2} = \frac{\omega}{c_S\sqrt{2}}\sqrt{\frac{1}{k_G} \pm \sqrt{\left(\frac{1}{k_G}\right)^2 + \frac{1}{Ck_G}\left(\frac{\pi c_S}{H\omega}\right)^2}} \tag{5}$$

In this equation, k_1 is real whatever the value of C and the frequency, corresponding therefore to a propagating wave, with an asymptotic limit when C tends to $+\infty$, i.e. for the shear beam case. Conversely, k_2 is always complex ($\frac{1}{k_G} < \sqrt{\left(\frac{1}{k_G}\right)^2 + \frac{1}{Ck_G}\left(\frac{\pi c_S}{H\omega}\right)^2}$)introducing a spatial attenuation of the propagating wave from the input location. This wave refers to an evanescent wave and Ebrahimian and Todorovska [5] showed that only the first solution wave is typically observed in buildings.

Finally, the phase velocity of the Timoshenko beam wave c_{TB} solution of Eq. (5) is:

$$c_{TB} = \frac{\omega}{k} = \frac{c_S\sqrt{2}}{\sqrt{\frac{1}{k_G} + \sqrt{\left(\frac{1}{k_G}\right)^2 + \frac{1}{Ck_G}\left(\frac{\pi c_S}{H\omega}\right)^2}}} \tag{6}$$

This velocity depends on the frequency (ω) and the structural properties (shear wave velocity c_S, dimensionless parameter C and height H).

In the pure shear beam case (i.e. $C \to +\infty$), the velocity does not depend on the frequency and equals the constant value $c_S\sqrt{k_G}$. In this pure shear case, its behavior is therefore not dispersive. Many discussions exist in the literature on the value of the shear correction factor k_G [27]. For a rectangular section, the shear correction factor can vary