Soil Dynamics and Earthquake Engineering 105 (2018) 22–26

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Dynamic interaction between a partially corroded pipeline and saturated poroelastic medium under plane waves

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ARTICLE INFO

Keywords:
Corroded pipeline
Saturated poroelastic medium
Plane waves
Dynamic stress

ABSTRACT

A fundamentally based mathematical model of a partially corroded pipeline in a saturated poroelastic medium is presented, and the dynamic interaction around the pipeline under plane waves is derived. Based on Biot's poroelastic theory, the dynamic governing equations of the saturated poroelastic medium are decomposed. The wave fields around the corroded pipeline are expanded by using the wave function expansion method. The loose poroelastic medium in the corroded areas is simulated by the spring-type medium, and the thickness of pipeline is small. By introducing the different boundary conditions in the corroded and perfect areas, the expanded coefficients are solved. Through numerical examples, the jump of dynamic stress resulting from the corroded area under different wave frequencies is examined in detail.

1. Introduction

Buried pipelines are important infrastructures, and go together with the daily lives of city dwellers and the industrial development. To improve the service performance of underground pipelines, the dynamic strength of buried pipelines subjected to various loadings is attracting more and more interests.

The fatigue of existing gas and liquid pipelines can bring many potential hazards to the people and environment. Corrosion defects are very common in the gas and liquid pipelines, and they may compromise the safety of pipelines. In recent years, several numerical [1,2], experimental [3–5] and analytical methods [6–8] have been used to investigate the serving behavior of corroded pipelines. By using the direct boundary element method, Stamos and Beskos investigated the dynamic response of infinite tunnels in an elastic or viscoelastic half-space subjected to seismic waves [1]. Based on Biot's theory, the diffraction of harmonic waves by tunnels in an infinite poroelastic saturated soil was studied numerically, and the effect of poroelasticity on the response was assessed [2]. Zeinoddini et al. reported the experimental result of residual stress measured on single/double and partial/full repaired welds in offshore pipelines [3]. The failure of gas transmission pipeline with protective coating in the underground was experimentally studied [4]. By using low strain rate measurement, Kentish studied the surface roughness effect on the stress resistance of corroded gas pipelines, and the orientation effect of test pieces on the stress was discussed [5]. Divino et al. constructed the Finite Element (FE) model of pipelines with corrosion defects to investigate the stress concentration [6]. The three-dimensional finite element model was introduced to analyze the serving performance of pipelines with repaired cracks, and the stress intensity factors were computed [7].

In the above reviewed papers, only the static loadings were studied. To deal with the 3D dynamic analysis of corroded pipelines, substantial computational cost is required, and it is impossible to accurately solve the 3D problems by FE methods. The analytical method is the most efficient way of predicting the response of corroded pipelines. For dynamic loadings, the analytical method is primarily developed to solve the dynamic behavior of pipelines with defects [8,9]. The dynamic interaction between the corroded pipelines and surrounding soil medium is a complicated contact problem, and the dynamic response to seismic waves is of high interest for the safe operation of pipelines. In the past, a famous fluid-saturated poroelastic medium was proposed by Biot [10] to describe the complex soil-water interaction. In the poroelastic medium, the corroded pipelines are very common. However, the dynamic response of corroded pipelines in poroelastic medium has not been dealt with.

To control the jump of dynamic stress and keep the structural integrity of oil and gas pipelines under acceptable risk level, closed-form solutions of displacements and dynamic stresses around a corroded pipeline in poroelastic medium under plane P and S waves are presented. This work concentrates on the dynamic interaction between the corroded pipelines and the surrounding soil medium due to the existence of corroded areas around the pipeline. In the corroded areas, the

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https://doi.org/10.1016/j.soildyn.2017.11.022
Received 6 November 2017; Received in revised form 12 November 2017; Accepted 12 November 2017

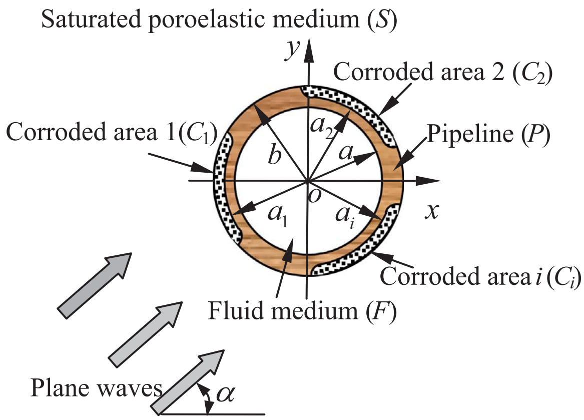

Fig. 1. A corroded pipeline in the saturated poroelastic medium.

loose soil medium is modeled as a distribution of springs. Some numerical examples are given and the jump of dynamic stress under different corroded conditions is discussed.

2. Governing equations and general solutions

Consider an infinite saturated poroelastic medium embedded with a partially corroded pipeline, as shown in Fig. 1. The circular pipeline is filled with fluid. Three regions are assumed, i.e., saturated poroelastic medium (*S*), pipeline (*P*) and fluid medium (*F*). The inner and outer radii of the pipeline are a and b, respectively. It is assumed that several corroded areas denoted by C_i ($i = 1, 2, 3, ...$) exist. Due to the corrosion, the radii of pipeline in these areas are smaller, and the radius of the *i*th corroded area is a_i. It is supposed that the incident slow or fast wave with frequency of ω propagates with angle α. The propagating direction is also perpendicular to the axis of pipeline.

2.1. General solutions of governing equations in poroelastic medium

The governing equations of solid material and the pore fluid are given as [10]

$$\mu_M u_{i,jj} + (\lambda_M + \alpha^2 M + \mu_M) u_{j,ji} + \alpha M w_{j,ji} = \rho_M \ddot{u}_i + \rho_F \ddot{w}_i, \tag{1}$$

$$\alpha M u_{j,ji} + M w_{j,ji} = \rho_F \ddot{u}_i + \frac{\rho_F}{\phi}\ddot{w}_i + \frac{\eta}{\kappa}\dot{w}_i, \quad i, j = x, y, \tag{2}$$

where u_i and w_i are the displacements of the solid medium and the infiltration displacements of the pore fluid, respectively. λ_M and μ_M are the Lamé constants of solid medium. ρ_M and ρ_F are the densities of porous medium and pore fluid. α and M are Biot's parameters. ϕ is the porosity. κ and η are, respectively, the permeability and the fluid viscosity.

In the saturated poroelastic medium, the constitutive relations of liquid-solid coupling field are written as

$$\sigma_{ij} = (\lambda_M e - \beta P_f)\delta_{ij} + 2\mu_M \varepsilon_{ij}, \tag{3}$$

$$P_f = M(\xi - \beta e), \quad e = u_{i,i}, \quad \xi = -w_{i,i}, \quad i, j = x, y, \tag{4}$$

where σ_{ij} are the components of stresses in the soil medium, e and ε_{ij} are, respectively, the dilatation and strain component of the solid matrix, ξ is the volume of fluid injection into unit volume of bulk material, P_f represents the excess pore pressure, δ_{ij} is the Kronecker delta.

In the present model, the dimension in the *z* direction is much larger than the other two, and the stresses acting normal to the section plane of pipeline are negligible, and the displacement in the *z* directions is negligible compared to the *x* and *y* directions. So, the three-dimensional model can be reduced to the two-dimensional one, i.e., plane strain model. For the plane strain problem, two scalar potentials φ_f, φ_s and one vector potential ψ are used, and the decoupling equations can be expressed as

$$\nabla^2\varphi_{f,s} + k_{f,s}^2\varphi_{f,s} = 0, \quad \nabla^2\psi + k_t^2\psi = 0, \tag{5}$$

where k_f, k_s, and k_t denote, respectively, the complex wave numbers of the fast compressional, slow compressional and shear waves. They are expressed as

$$k_{f,s}^2 = \frac{\beta_1 A_{f,s} - \beta_2}{A_{f,s}}, \; k_t^2 = \beta_3/\mu, \tag{6}$$

where $\beta_1 = -\beta/M$, $\beta_2 = \alpha\beta + \rho_f\omega^2$, $\beta_3 = \alpha - \rho_F\omega^2/\beta$, and $A_{f,s}$ satisfies the following equation

$$A_{f,s}^2 + \frac{\beta_3 - (\lambda_M + 2\mu_M)\beta_1 - \beta_2\beta_4}{\beta_1\beta_4}A_{f,s} + \frac{(\lambda_M + 2\mu_M)\beta_2}{\beta_1\beta_4} = 0. \tag{7}$$

The displacement, the pore pressure and the stresses can be expressed as follows

$$u_r = \frac{\partial\varphi_f}{\partial r} + \frac{\partial\varphi_s}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}, \; u_\theta = \frac{1}{r}\left(\frac{\partial\varphi_f}{\partial\theta} + \frac{\partial\varphi_s}{\partial\theta}\right) - \frac{\partial\psi}{\partial r}, \; w_r = \frac{1}{\gamma}\frac{\partial p_f}{\partial r} - \frac{\rho_f\omega^2}{\gamma}u_r,$$
$$P_f = -A_f k_f^2\varphi_f - A_s k_s^2\varphi_s, \tag{8}$$

$$\sigma_{rr} = \frac{1}{r^2}\left\{-2\mu_M\frac{\partial\psi}{\partial\theta} + \lambda_M\left[\frac{\partial^2\varphi_f}{\partial\theta^2} + \frac{\partial^2\varphi_s}{\partial\theta^2} + r\left(\frac{\partial\varphi_f}{\partial r} + \frac{\partial\varphi_s}{\partial r}\right)\right] + r\left[2\mu_M\frac{\partial^2\psi}{\partial r\partial\theta} + r(\lambda_M + 2\mu_M)\left(\frac{\partial^2\varphi_f}{\partial r^2} + \frac{\partial^2\varphi_s}{\partial r^2}\right)\right]\right\} - \alpha p_f, \tag{9}$$

$$\sigma_{\theta\theta} = \frac{1}{r^2}\left\{-2\mu_M\frac{\partial\psi}{\partial\theta} + \frac{\partial^2\varphi_f}{\partial\theta^2} + \frac{\partial^2\varphi_s}{\partial\theta^2} + r(\lambda_M + 2\mu_M)\left(\frac{\partial^2\varphi_f}{\partial r^2} + \frac{\partial^2\varphi_s}{\partial r^2}\right) - 2r\mu_M\frac{\partial^2\psi}{\partial r\partial\theta} + \lambda_M\left[\frac{\partial^2\varphi_f}{\partial\theta^2} + \frac{\partial^2\varphi_s}{\partial\theta^2} + r^2\left(\frac{\partial^2\varphi_f}{\partial r^2} + \frac{\partial^2\varphi_s}{\partial r^2}\right)\right]\right\} - \alpha p_f, \tag{10}$$

$$\sigma_{r\theta} = \frac{\mu_M}{r^2}\left\{-2\frac{\partial\varphi_f}{\partial\theta} - 2\frac{\partial\varphi_s}{\partial\theta} + \frac{\partial^2\psi}{\partial\theta^2} + r\left[\frac{\partial\psi}{\partial r} + 2\left(\frac{\partial^2\varphi_f}{\partial r\partial\theta} + \frac{\partial^2\varphi_s}{\partial r\partial\theta}\right) - r\frac{\partial^2\psi}{\partial r^2}\right]\right\}. \tag{11}$$

From Eqs. (6) and (7), the total wave field in the solid medium is the superposition of the incident wave field and the scattered wave field, i.e.,

$$\varphi_f^{(M)} = \varphi_0^f\sum_{n=-\infty}^{\infty}[i^n J_n(k_f r)e^{in(\theta-\alpha)} + A_n H_n^{(1)}(k_f r)e^{in\theta}]e^{i\omega t}, \tag{12}$$

$$\varphi_s^{(M)} = \varphi_0^s\sum_{n=-\infty}^{\infty}[i^n J_n(k_s r)e^{in(\theta-\alpha)} + B_n H_n^{(1)}(k_s r)e^{in\theta}]e^{i\omega t}, \tag{13}$$

$$\varphi_s^{(M)} = \psi_0\sum_{n=-\infty}^{\infty}[i^n J_n(k_t r)e^{in(\theta-\alpha)} + C_n H_n^{(1)}(k_t r)e^{in\theta}]e^{i\omega t}, \tag{14}$$

where $J_n(\bullet)$ denotes the *n*th Bessel function of the first kind, $H_n^{(1)}(\bullet)$ denotes the *n*th Hankel function of the first kind, $\varphi_0^{f,s}$ and ψ_0 are the amplitudes of P and S waves, A_n, B_n and C_n are the expanded mode coefficients.